Es 1

$$\int_0^{\pi/8} \frac{\operatorname{sen}^2 x \quad dx}{|\ln(\cos x)|^\alpha \cos(2x)} = I, \quad \cos x \neq 0,\ \cos(2x) \neq 0 \quad \forall x \in [0, \tfrac{\pi}{8}]$$

$$\Rightarrow I \sim \int_0^{\pi/8} \frac{(x+o(x))^2 \quad dx}{\left|\ln\left(1-\frac{x^2}{2}+o(x^2)\right)\right|^\alpha} \sim \int_0^{\pi/8} \frac{x^2}{\left(\frac{x^2}{2}\right)^\alpha} dx \sim \int_0^{\pi/8} \frac{dx}{x^{(2\alpha-2)}}$$

convergente $\Leftrightarrow 2\alpha - 2 < 1 \Leftrightarrow 2\alpha < 3 \Leftrightarrow \alpha < 3/2$

$$\Rightarrow I \text{ è } \begin{cases} \text{convergente se } \alpha < 3/2 \\ \text{divergente a } +\infty \quad \text{se } \alpha \geq \frac{3}{2} \end{cases}$$

Es 2

- $\cos(\arctan(x)) - \cos x = 1 - \frac{(\arctan(x))^2}{2} + \frac{(\arctan(x))^4}{24} + o(x^4)$

$$- 1 + \frac{x^2}{2} - \frac{x^4}{24} + o(x^4) =$$

$$= \cancel{1} - \frac{\left(x - \frac{x^3}{3} + o(x^3)\right)^2}{2} + \frac{(x+o(x))^4}{24} - \cancel{1} + \frac{x^2}{2} - \frac{x^4}{24} + o(x^4) =$$

$$= \cancel{-\frac{x^2}{2}} + \frac{x^4}{3} + \cancel{\frac{x^4}{24}} + \cancel{\frac{x^2}{2}} - \cancel{\frac{x^4}{24}} + o(x^4) = \boxed{\frac{x^4}{3} + o(x^4)}$$

- $\ln(1+x^2) - \operatorname{sen}(x^2) = x^2 - \frac{x^4}{2} + o(x^4) - x^2 + o(x^4) = \boxed{-\frac{x^4}{2} + o(x^4)}$

$$\Rightarrow \ell = \lim_{x\to 0} \frac{\cos(\arctan(x)) - \cos x}{\ln(1+x^2) - \operatorname{sen}(x^2)} = \lim_{x \to 0} \frac{\frac{x^4}{3} + o(x^4)}{-\frac{x^4}{2} + o(x^4)} = \boxed{-\frac{2}{3}}$$

Es. 3 $\quad f(x) = \arctan\left(\left|\frac{x-1}{x+1}\right|\right)$

i) $\operatorname{Dom}(f) = \mathbb{R} \setminus \{-1\}$

ii) $\lim_{x \to -1} f(x) = \frac{\pi}{2}$, $\lim_{x \to \pm\infty} f(x) = \frac{\pi}{4} \Rightarrow$ $\boxed{y = \pi/4 \text{ è asintoto orizzontale bilatero}}$

iii) $f'(x) = \frac{1}{1+x^2} \operatorname{sgn}\left(\frac{x-1}{x+1}\right) \geq 0 \iff x \in]-\infty,-1[\cup [1,+\infty[$

$\Rightarrow f$ crescente su $]-\infty,-1[$ e su $[1,+\infty[$, f decrescente su $]-1,1]$

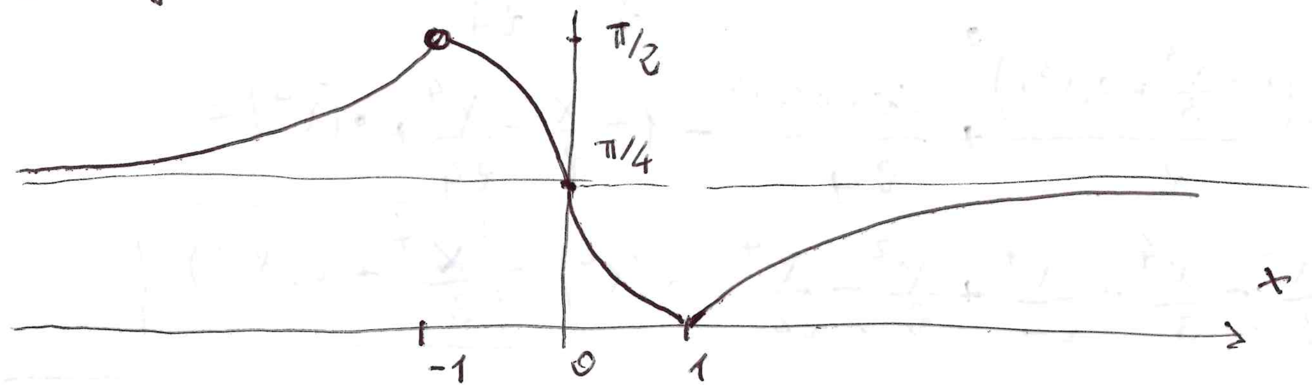
iv) $x=1$ punto di minimo assoluto, $\nexists$ punti di massimo relativo ed assoluto, $0 = f(1) = \min f$

v) $f''(x) = \frac{-2x}{(1+x^2)^2} \operatorname{sgn}\left(\frac{x-1}{x+1}\right) \geq 0 \iff x \in]-\infty,-1[\cup [0,1]$

$\Rightarrow f$ è convessa su $]-\infty,-1[$ e su $[0,1]$

f è concava su $]-1,0]$ e su $[1,+\infty[$, $x=0$ è punto di flesso

vi) $\operatorname{Im}(f) = [0, \pi/2[$



$\boxed{\text{Es. 4}}$ $f(x) = \int_{x^2}^{2x^2} \frac{e^{-t}-1}{t}\,dt$, i) $\lim_{t\to 0} \frac{e^{-t}-1}{t} = -1 \Rightarrow \operatorname{Dom}(f) = \mathbb{R}$

$\frac{e^{-t}-1}{t} < 0 \quad \forall t>0 \Rightarrow f(x) \leq 0 \quad \forall x \in \mathbb{R} \Rightarrow \{f \leq 0\} = \mathbb{R}$

ii) $f'(x) = \frac{e^{-2x^2}-1}{2x^2} \cdot 4x - \left(\frac{e^{-x^2}-1}{x^2}\right) \cdot 2x = \frac{2}{x}\left[e^{-2x^2} - 1 - e^{-x^2} + 1\right]$

$= \frac{2}{x} e^{-x^2}\left(e^{-x^2}-1\right) \geq 0 \iff x < 0 \Rightarrow$

f crescente su $]-\infty,0]$, f decrescente su $[0,+\infty[$

$\boxed{\text{Es. 5}}$ $(1+\operatorname{sen}^2 x)\cdot y' = \cos x \left[y + (1+\operatorname{sen}^2 x)\, e^{\arctan(\operatorname{sen} x)}\right]$

$$\Rightarrow y' = \frac{\cos x}{(1+\operatorname{sen}^2 x)}\cdot y + \cos x \cdot e^{\arctan(\operatorname{sen} x)} \qquad (1)$$

i) Eq omogenea associata ad (1):

$$y' = \frac{\cos x}{1+\operatorname{sen}^2 x}\cdot y \qquad (2)$$

$\displaystyle\int \frac{\cos x}{1+\operatorname{sen}^2 x}\,dx = \arctan(\operatorname{sen} x) + c \Rightarrow$ Integrale generale di (2) è: $\boxed{\varphi_c(x) = c\, e^{\arctan(\operatorname{sen} x)}, \quad c \in \mathbb{R}}$

ii) Eq. completa (1):

$$\int e^{-\arctan(\operatorname{sen} x)} \cdot e^{\arctan(\operatorname{sen} x)} \cdot \cos x \overset{dx}{=} \int \cos x \, dx =$$

$= \operatorname{sen} x + c \Rightarrow$ Integrale generale di (1) è:

$$\boxed{\varphi_c(x) = e^{\arctan(\operatorname{sen} x)}\left(c + \operatorname{sen} x\right)}$$

iii) Pb Cauchy, $y\left(\frac{\pi}{2}\right) = 0 \Rightarrow 0 = \varphi_c\left(\frac{\pi}{2}\right) = e^{\pi/4}(c+1)$

$\Rightarrow c = -1 \Rightarrow$ Sol. del Pb di Cauchy è:

$$\boxed{\varphi(x) = e^{\arctan(\operatorname{sen} x)}\left(\operatorname{sen} x - 1\right)}$$