$\boxed{\text{Es 1}}$ $\displaystyle\int_0^{\pi/8} \frac{|\ln(\cos x)|^{\alpha}}{\operatorname{sen}^3(x)\cdot\cos(2x)}\,dx = I$, $\operatorname{sen} x \neq 0$, $\cos(2x) \neq 0$ $\forall x \in]0, \frac{\pi}{8}]$

$$\Rightarrow I \sim \int_0^{\pi/8} \frac{|\ln(1-\frac{x^2}{2}+o(x^2))|^{\alpha}}{(x+o(x))^3}\,dx \sim \int_0^{\pi/8} \frac{\left(\frac{x^2}{2}\right)^{\alpha}}{x^3}\,dx \sim \int_0^{\pi/8} \frac{1}{x^{(3-2\alpha)}}\,dx$$

convergente $\Leftrightarrow 3-2\alpha < 1 \Leftrightarrow 2\alpha > 2 \Leftrightarrow \alpha > 1$

$$\Rightarrow I \text{ è } \begin{cases} \text{convergente} & \text{se } \alpha > 1 \\ \text{divergente a } +\infty & \text{se } \alpha \leq 1 \end{cases}$$

$\boxed{\text{Es.2}}$

- $\arctan(x^2) - \ln(1+x^2) = x^2 + o(x^5) - x^2 + \frac{x^4}{2} + o(x^4) = \boxed{\frac{x^4}{2}+o(x^4)}$

- $\cos(\tan(x)) - \cos x = \cancel{1} - \frac{(\tan(x))^2}{2} + \frac{(\tan(x))^4}{24} + o(x^4) - \cancel{1} + \frac{x^2}{2} - \frac{x^4}{24} + o(x^4) =$

$$= -\frac{\left(x+\frac{x^3}{3}+o(x^3)\right)^2}{2} + \frac{(x+o(x))^4}{24} + \frac{x^2}{2} - \frac{x^4}{24} + o(x^4) = \cancel{-\frac{x^2}{2}} - \frac{x^4}{3} + \cancel{\frac{x^4}{24}} + \cancel{\frac{x^2}{2}} - \cancel{\frac{x^4}{24}} + o(x^4)$$

$$= \boxed{-\frac{x^4}{3}+o(x^4)} \Rightarrow \ell = \lim_{x\to 0} \frac{\arctan(x^2)-\ln(1+x^2)}{\cos(\tan(x))-\cos x} = \lim_{x\to 0} \frac{\frac{x^4}{2}+o(x^4)}{-\frac{x^4}{3}+o(x^4)} = \boxed{-\frac{3}{2}}$$

$\boxed{\text{Es. 3}}$ $f(x) = \operatorname{arccot}\left(\left|\frac{x+2}{x-2}\right|\right)$

i) $\operatorname{Dom}(f) = \mathbb{R} \setminus \{2\}$

ii) $\displaystyle\lim_{x\to 2} f(x) = 0$, $\displaystyle\lim_{x\to\pm\infty} f(x) = \frac{\pi}{4} \Rightarrow$ $\boxed{y = \frac{\pi}{4} \text{ è asintoto orizzontale bilatero}}$

iii) $f'(x) = \dfrac{2}{x^2+4}\operatorname{sgn}\left(\dfrac{x+2}{x-2}\right) \geq 0 \Leftrightarrow x \in]-\infty,-2] \cup]2,+\infty[$

$\Rightarrow f$ crescente su $]-\infty,-2]$ e su $]2,+\infty[$, f decrescente su $[-2,2[$

iv) $x=-2$ punto di massimo assoluto, $\frac{\pi}{2} = f(-2) = \max f$

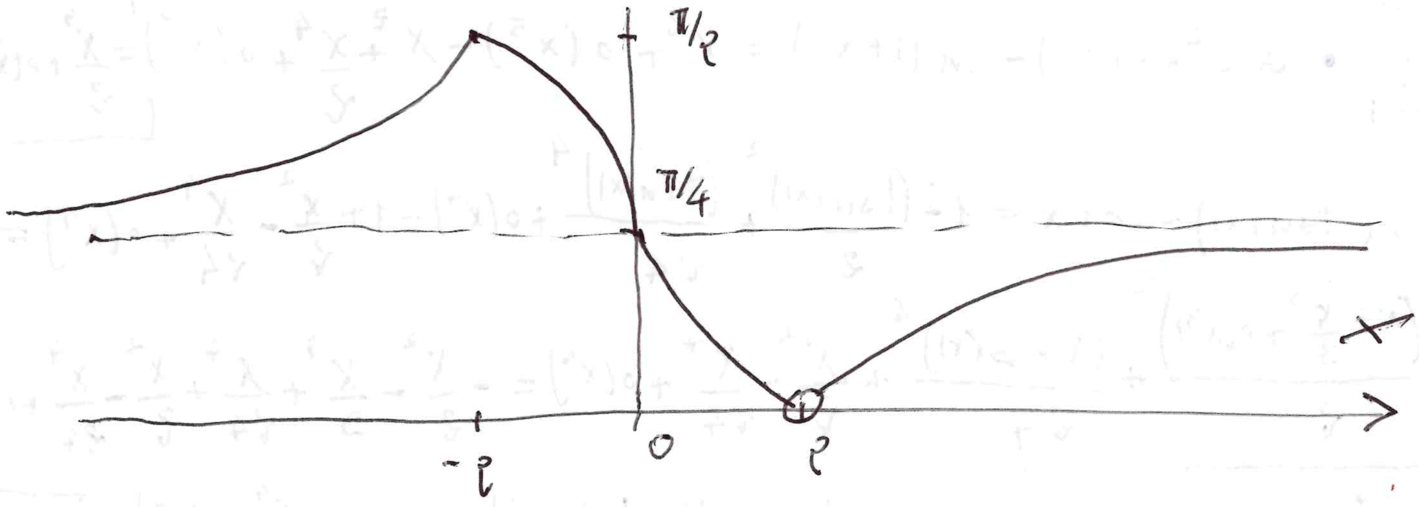
$\nexists$ punti di minimo relativo od assoluto

v) $f''(x) = \frac{-\cancel{4}x}{(x^2+4)^2} \, \mathrm{sgn}\left(\frac{x+2}{x-2}\right) \geq 0 \iff x \in \,]-\infty, -2] \cup [0, 2[$

$\Rightarrow$ f è convessa su $]-\infty, -2]$ e su $[0, 2[$ $\quad$ $x=0$ è punto di flesso

f è concava su $[-2, 0]$ e su $]2, +\infty[$

vi) $\mathrm{Im}(f) = \,]0, \frac{\pi}{2}]$



$\boxed{\text{Es.4}}$ $f(x) = \int_{x^2}^{3x^2} \frac{e^t - 1}{t} \, dt$

i) $\lim_{t \to 0} \frac{e^t - 1}{t} = 1 \Rightarrow \mathrm{Dom}(f) = \mathbb{R}$

$\frac{e^t - 1}{t} > 0 \quad \forall t > 0 \Rightarrow f(x) \geq 0 \quad \forall x \in \mathbb{R} \Rightarrow \{f \geq 0\} = \mathbb{R}$

ii) $f'(x) = \frac{e^{3x^2} - 1}{3x^2} \cdot 6x - \left(\frac{e^{x^2} - 1}{x^2}\right) \cdot 2x = \frac{2}{x}\left[e^{3x^2} - 1 - e^{x^2} + 1\right] =$

$= \frac{2e^{x^2}}{x}\left(e^{2x^2} - 1\right) \geq 0 \iff x > 0 \Rightarrow$ $\boxed{x=0 \text{ punto di minimo assoluto}}$

f crescente su $[0, +\infty[$, f decrescente su $]-\infty, 0]$ $\Uparrow$

$\boxed{\text{Es. 5}}$ $(1+\cos^2 x)\, y' = \operatorname{sen} x \left[(1+\cos^2 x)\, e^{\arctan(\cos x)} - y\right]$

$$\Rightarrow y' = \frac{-\operatorname{sen} x}{(1+\cos^2 x)} \cdot y + \operatorname{sen} x \cdot e^{\arctan(\cos x)} \qquad (1)$$

i) Eq. omogenea associata a (1):

$$y' = \frac{-\operatorname{sen} x}{1+\cos^2 x} \cdot y \qquad (2)$$

$\int \frac{-\operatorname{sen} x}{1+\cos^2 x}\, dx = \arctan(\cos x) + c \Rightarrow$ Integrale generale di (2) è:

$$\boxed{\varphi_c(x) = c\, e^{\arctan(\cos x)}, \quad c \in \mathbb{R}}$$

ii) Eq. completa (1):

$$\int e^{-\arctan(\cos x)} \cdot e^{\arctan(\cos x)} \cdot \operatorname{sen} x \, dx = \int \operatorname{sen} x \, dx = -\cos x + c$$

$\Rightarrow$ Integrale generale di (1) è:

$$\boxed{\varphi_c(x) = e^{\arctan(\cos x)} \left(c - \cos x\right)}$$

iii) Pb Cauchy, $y(0) = 0 \Rightarrow 0 = \varphi_c(0) = e^{\pi/4}(c-1) \Rightarrow$

$c = 1 \Rightarrow$ Sol. del Pb di Cauchy è:

$$\boxed{\varphi(x) = e^{\arctan(\cos x)} (1 - \cos x)}$$