Subdivision surfaces for CAD: integration through parameterization and local correction

Michele Antonelli\textsuperscript{1}, Carolina Beccari\textsuperscript{2}, Giulio Casciola\textsuperscript{2}, Serena Morigi\textsuperscript{2}

\textsuperscript{1}Department of Mathematics, University of Padova, Italy
\textsuperscript{2}Department of Mathematics, University of Bologna, Italy

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Subdivision surfaces and CAD

SS widely supported in nearly all modeling programs

Advantages: flexibility for arbitrary topology + superset of NURBS “standard”

A lot of theoretical study and many proposed algorithms potentially useful in CAD:

- surface fitting (Ma and Zhao, 2002)
- reverse engineering (Ma and Zhao, 2000; Beccari, Farella, Liverani, Morigi, and Rucci, 2010)
- curve lofting (Nasri, 2001; Schaefer, Warren, and Zorin, 2004)

Their presence in CAD is still negligible:

- no closed-form representation
- quality and regularity issues

Subdivision surfaces integrated in a CAD system

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2010–2013: Eurostars Project NIIT4CAD
(New Interactive and Innovative Technologies for CAD)

A., Beccari, Casciola, Ciarloni, Morigi: Subdivision surfaces integrated in a CAD system, CAD 45(11), 2013
Objective

• Seamless integration of subdivision surfaces in a CAD system
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Catmull-Clark

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- Main roadblocks:
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  - difficulty of integration into the modeling workflow
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Seamless integration

• The desired **accuracy** is achieved
• All the **functionalities** of the CAD system are inherited
SS seamlessly integrated in our system
SS seamlessly integrated in our system

Mean curvature

< -0.2035 > 0.07465

Catmull-Clark

Local correction

Isophotes

< -0.2053 > 0.07465

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Subdivision surfaces integrated in a CAD system

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1. Features that a CAD system must possess to allow the integration of SS
2. Integration of subdivision solids through parameterization and boundary representation
3. Local correction of regularity issues
4. Examples
The CAD system paradigm

- Geometric kernel:
  - set of geometric representations (NURBS, planes, cylinders, quadrics)
  - set of tools which operate on them (intersections, projections, Boolean operations, offsets, fillets, etc.)

- Parametric curves and surfaces
  Solids: B-rep

- Heterogeneous geometric description

- Extensible geometric kernel
The CAD system paradigm

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  - set of tools which operate on them (intersections, projections, Boolean operations, offsets, fillets, etc.)

- **Parametric curves and surfaces**

  **Solids:** B-rep

  **Boundary representation:**
a geometric model is described through its geometric limits, storing information on topology and geometry.

  - **Solid:** volume limited by **shells** (boundary solid/non-solid)
  - **Shell:** collection of surface patches, called B-rep **faces**
  - The boundary of a **face** is composed of **loops** of connected **edges**
  - **Vertex:** limit of an **edge**

The B-rep induces a structure of graph between the different components.

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Subd B-rep

A Subd B-rep represents a B-rep geometric model in which:

- each B-rep face is a subdivision surface patch associated with a rectangular parametric domain;
- the B-rep topology is inferred from the subdivision control mesh.
Integration

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**Idea:** the Subd B-rep should maintain an intuitive association (possibly 1-1) between the control mesh faces and the B-rep faces.
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- **Topology:**
  - Control mesh: 1 quad face, 1 n-sided face
  - Subd B-rep: 1 face, $n$ quad faces

- **Geometry:**
  - Control mesh: 1 face
  - Subd B-rep: 1 parametric quad patch on base domain $Q := [0,1]^2$
B-rep geometry description \[1\]

- 1 face $\leftrightarrow$ 1 parametric quad patch $S_i$ on $Q$
- For Catmull-Clark surfaces, we are able to evaluate two types of patches:
  - regular (bi-cubic tensor-product B-splines)
  - quadrilateral and containing a single extraordinary vertex
- parameterization function $\psi_{S_i}: Q \to S_i$

Limit surface structure around an extraordinary vertex

Stam, 1998; Yamaguchi, 2001; Lai and Cheng, 2006
B-rep geometry description [1]

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Limit surface structure around an extraordinary vertex

(a) Quad face with a single e.v.
(b) Quad face with more than one e.v.

\[
\begin{align*}
\Phi_{\ell} &= \sigma_2 \circ \rho^{-1}_{\ell \frac{\pi}{2}} \circ \tau_{q_{\ell}-q_0} \\
\sigma_h &= \text{scaling by a factor } h \\
\rho_a^{-1} &= \text{c.w. rotation around the origin of an angle } a \\
\tau_v &= \text{translation of a vector } v
\end{align*}
\]
(b) Quad face with more than one e.v.

(c.1) Non-quad face without e.v.

(c.2) Non-quad face with at least one e.v.
Surface tuning

Objective: local correction of quality issues
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• The shape of CC surfaces is satisfactory $\Rightarrow$ we are interested in maintaining their appearance and B-spline nature in the widest possible area, while tuning their analytical properties in the smallest neighborhood of e.v.
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Idea: local correction through polynomial blending (A. Levin, 2006; Zorin, 2006)

\[ S_i^* := w S_i + (1 - w) P \]

blended surface

Catmull-Clark surface

weight function

approximating polynomial

Catmull-Clark

Local correction

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**Surface tuning**

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- \( w \) s.t. \( C^2 \)-transition between \( S_i^* \) and \( S_i \)

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Approach: definition of a parametric surface evaluable at arbitrary points
- Levin, Zorin: discrete vs. Our: parametric surface with exact evaluation

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A common parameterization domain

- $S_i, P, w$ must be parameterized over a common domain
A common parameterization domain

- \( S_i, P, w \) must be parameterized over a common domain
  \( \implies \) characteristic map of valence \( n \), regarded as a parametric multipatch surface:
  \[ \psi_{K_0^{[n]}} : Q \rightarrow K_0^{[n]} \]
  one sector
A common parameterization domain

- $S_i, P, w$ must be parameterized over a common domain
  $\Rightarrow$ characteristic map of valence $n$, regarded as a parametric multipatch surface: $\psi_{K_0^{[n]}} : Q \rightarrow K_0^{[n]}$

- **Star-shaped transformation**
  $K_i := \rho_i \frac{2\pi}{n} \circ \psi_{K_0^{[n]}} \circ \sigma_h \circ \phi$
  - $\phi(u, v) := \begin{cases} (u, v) & \text{if patch of type (a) or (c.1)} \\ \phi_\ell(u, v) & \text{if patch of type (b) or (c.2)} \end{cases}$
  - $\sigma_h$ scaling of $h$
  - $\psi_{K_0^{[n]}} : Q^h \mid_Q \rightarrow K_0^{[n]}$, $Q^h := [0, h]^2$
  - $\rho_i \frac{2\pi}{n}$ c.c.w. rotation of angle $i \frac{2\pi}{n}$

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  $\Rightarrow$ characteristic map of valence $n$, regarded as a parametric multipatch surface: $\psi_{K_0^{[n]}} : Q \rightarrow K_0^{[n]}$ one sector

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  $\kappa_i := \rho_i \frac{2\pi}{n} \circ \psi_{K_0^{[n]}} \circ \sigma_h \circ \phi$
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  - $\sigma_h$ scaling of $h$
    
    | patch type | (a) | (b) | (c.1) | (c.2) |
    |------------|-----|-----|-------|-------|
    | $h$        | 4   | 2   | 2     | 1     |
  - $\psi_{K_0^{[n]}} : Q^h|_Q \rightarrow K_0^{[n]}$, $Q^h := [0,h]^2$
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- Star-shaped domain
  $K_n := \bigcup_{i=0}^{n-1} \{ \kappa_i(u,v) \mid (u,v) \in Q \text{ and } \sigma_h(\phi(u,v)) \in Q \}$

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- $S_i, P, w$ must be parameterized over a common domain $\mapsto$ characteristic map of valence $n$, regarded as a parametric multipatch surface: $\psi_{K_0^{[n]}} : Q \rightarrow K_0^{[n]}$

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- **Blending region**
  $D_n := \left\{ (s, t) \in K_n \big| \| (s, t) \|_2 \leq \lambda^{[n]} \right\}$
  $(s, t) := \kappa_i(u, v), \ \lambda^{[n]}$ subdominant eigenvalue of the subdivision matrix

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• **Blended surface** \( S_i^*(u,v) := \begin{cases} 
 w(s,t)S_i(u,v) + (1-w(s,t))P(s,t) & (s,t) \in D_n \\
 S_i(u,v) & \text{elsewhere} \end{cases} \) 

where \((s,t) := \kappa_i(u,v)\)
Blended surface on $K_n$

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  where $(s,t) := \kappa_i(u,v)$

- **Preimage of $D_n$ under $\kappa_i$**

  $\tilde{D}_{n,i} := \left\{ (u,v) \in Q \mid \sigma_h(\phi(u,v)) \in Q \text{ and } \|\kappa_i(u,v)\|_2 \leq \lambda^n \right\}$

  $\tilde{D}_{n,i} \subset \begin{cases} Q^{1/3} & \text{if patch of type (a) or (b)} \\ Q^{1/4} & \text{if patch of type (c.1) or (c.2)} \end{cases}$

  $\implies$ blending regions surrounding e.v. of the same face are well separated

  $\implies$ most of the surface is spline!
LS approximating polynomial

- Interpolate the e.v. \( \mathbf{P}(s,t) = \mathbf{p}_{ev} + C \mathbf{m}(s,t), \quad \mathbf{m}(s,t) = (s, t, s^2, st, t^2, \ldots) \)

- Coefficients \( C \) are computed by least squares fitting

12 uniformly distributed approximation points per sector in \( \tilde{D}_{n,i} \subset Q \)
LS approximating polynomial

- Interpolate the e.v. \( P(s,t) = p_{ev} + Cm(s,t), \quad m(s,t) = (s,t,s^2,st,t^2,...) \)

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12 uniformly distributed approximation points per sector in \( \bar{D}_{n,i} \subset Q \)

- \( V^T V c = V^T (p - p_{ev}) \implies \) precompute and store \( (V^T V)^{-1} V^T \) for each valence \( n \)
**LS approximating polynomial**

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- Coefficients \( C \) are computed by least squares fitting

\[
\begin{align*}
12 \text{ uniformly distributed approximation points} \\
\text{per sector in } \tilde{D}_{n,i} \subset Q
\end{align*}
\]

- \( V^T V \mathbf{c} = V^T (\mathbf{p} - \mathbf{p}_{ev}) \implies \text{precompute and store } (V^T V)^{-1} V^T \text{ for each valence } n \)

- E.v. on boundary: *fan-shaped domain*

\[
\hat{K}_n := \bigcup_{i=0}^{n-2} \left\{ \hat{\psi}_{\hat{K}_i}[n] (\sigma_h(\phi(u,v))) \mid (u,v) \in Q \text{ and } \sigma_h(\phi(u,v)) \in Q \right\}
\]
Handling of heterogeneous rep.

- **Workflow** for the creation and editing of a Subd B-rep

![Diagram showing the workflow and components](image)
Handling of heterogeneous rep.

- **Workflow** for the creation and editing of a Subd B-rep

  ![Workflow Diagram]

  - Subd control mesh → Patcher → Subd-B-rep

- **Operations of solid composition** → B-rep whose faces can have *heterogeneous nature* (NURBS + subdivision) and are *editable* while maintaining this feature

  → the workflow applies to those faces of the heterogeneous B-rep model that are of *subdivision* type
Validation from Integration between conceptual design and engineering phase

Subd B-rep solid

Model with thickness

Division of the object in two parts

Subdivision surfaces integrated in a CAD system

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Thank you!