

Esercizi 3.

22/10/2010

Esercizio 1

Determinare la misura in radianti dei seguenti angoli espressi in gradi.

$$(a) \quad 240^\circ; 135^\circ; 330^\circ.$$

Esercizio 2

Calcolare i seguenti valori:

$$(a) \quad \cos\left(\frac{4}{3}\pi\right); \sin\left(\frac{3}{4}\pi\right); \tan\left(\frac{11}{6}\pi\right).$$

Esercizio 3

Verificare le seguenti identità.

$$(a) \quad 1 + \operatorname{tg}^2(\alpha) = \frac{1}{\cos^2(\alpha)}$$

$$(b) \quad 1 + \operatorname{ctg}^2(\alpha) = \frac{1}{\sin^2(\alpha)}$$

$$(c) \quad \sin^4(\alpha) - \cos^4(\alpha) = \sin^2(\alpha) - \cos^2(\alpha)$$

Esercizio 4

Risolvere le seguenti equazioni:

$$(a) \quad \sin(2x) = 1$$

$$(b) \quad 1 - \sqrt{2} \cdot \cos(x) = 0$$

$$(c) \quad \cos(\lambda x) = 1 \quad \text{supporre } \lambda > 0$$

$$(d) \quad \sin(3x) = 3$$

$$(e) \quad \sin^2(x) = \frac{1}{2}$$

$$(f) \quad \cos^2(x) - 3\cos(x) - 4 = 0$$

$$(g) \quad 2\sin^2(x) - \sin(x) = 1$$

$$(h) \quad \cos(x) + 1 = 2\sin^2(x)$$

Esercizio 5

Risolvere le seguenti disequazioni

$$(a) \quad 2 \cos(x) < \sqrt{3}$$

$$(b) \quad \sin^2(x) < \frac{3}{4}$$

$$(c) \quad |\cos(x)|^2 > \frac{3}{4}$$

SOLUZIONI

Esercizio 1

$$(a) \quad \frac{4}{3}\pi; \frac{3}{4}\pi; \frac{11}{6}\pi.$$

Esercizio 2

$$(a) \quad -\frac{1}{2}; \frac{\sqrt{2}}{2}; -\frac{\sqrt{3}}{3}.$$

Esercizio 4

$$a) \quad \left\{ \frac{\pi}{4} + k\pi : k \in \mathbb{Z} \right\}$$

$$b) \quad \left\{ \frac{\pi}{4} + 2k\pi : k \in \mathbb{Z} \right\} \cup \left\{ -\frac{\pi}{4} + 2k\pi : k \in \mathbb{Z} \right\}$$

$$c) \quad \left\{ \frac{2k\pi}{\lambda} : k \in \mathbb{Z} \right\}$$

$$d) \quad \emptyset$$

$$e) \quad \left\{ \frac{\pi}{4} + \frac{k\pi}{2} : k \in \mathbb{Z} \right\}$$

$$f) \quad \left\{ \pi + 2k\pi : k \in \mathbb{Z} \right\}$$

$$g) \quad \left\{ \frac{\pi}{2} + 2k\pi : k \in \mathbb{Z} \right\} \cup \left\{ \frac{7\pi}{6} + 2k\pi : k \in \mathbb{Z} \right\} \cup \left\{ \frac{11\pi}{6} + 2k\pi : k \in \mathbb{Z} \right\}$$

$$h) \quad \left\{ \pi + 2k\pi : k \in \mathbb{Z} \right\} \cup \left\{ \frac{\pi}{3} + 2k\pi : k \in \mathbb{Z} \right\} \cup \left\{ \frac{5\pi}{3} + 2k\pi : k \in \mathbb{Z} \right\}$$

Esercizio 5

$$a) \quad \cup_{k \in \mathbb{Z}} \left(\frac{\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi \right)$$

$$b) \quad \cup_{k \in \mathbb{Z}} \left(-\frac{\pi}{3} + k\pi, \frac{\pi}{3} + k\pi \right)$$

$$c) \quad \cup_{k \in \mathbb{Z}} \left(-\frac{\pi}{6} + k\pi, \frac{\pi}{6} + k\pi \right)$$