

Esercizi 5.

5/11/2010

Esercizio 1

Indicare (se vi sono) i punti di discontinuità delle seguenti funzioni.

$$(a) \quad f(x) = \begin{cases} 0 & x \leq 0 \\ 1+x & x > 0 \end{cases}$$

$$(b) \quad f(x) = \begin{cases} 0 & x \leq 0 \\ \text{sen}(x) & 0 < x \leq \pi \\ \cos(x) & \pi < x \end{cases}$$

$$(c) \quad f(x) = \begin{cases} e^{-x} & x < -1 \\ x+1 & -1 \leq x \leq 0 \\ x^2 & 0 < x \leq 1 \\ x & 1 < x \end{cases}$$

$$(d) \quad f(x) = \begin{cases} \text{sen}(x) & x \leq 0 \\ \text{tg}(x) & 0 < x < \frac{\pi}{2} \\ \cos(x) & \frac{\pi}{2} \leq x \end{cases}$$

$$(e) \quad f(x) = \begin{cases} \cos(x) & x \leq -\frac{\pi}{4} \\ \text{sen}(x) & -\frac{\pi}{4} < x < \frac{\pi}{4} \\ \cos(x) & \frac{\pi}{4} \leq x \end{cases}$$

$$(f) \quad f(x) = \begin{cases} \frac{x^2-4}{x-2} & x \neq 2 \\ 4 & x = 2 \end{cases}$$

Esercizio 2

Per quali valori di a e b le seguenti funzioni sono continue.

$$(a) \quad f(x) = \begin{cases} a & x \leq 0 \\ \cos(x) & x > 0 \end{cases}$$

$$(b) \quad f(x) = \begin{cases} \frac{\text{sen}(x)}{x} & x \neq 0 \\ a & x = 0 \end{cases}$$

$$(c) \quad f(x) = \begin{cases} e^{-x} & x < 0 \\ a+1 & x = 0 \\ be^x & x > 0 \end{cases}$$

$$(d) \quad f(x) = \begin{cases} \cos(x) & x \leq 0 \\ a+bx & 0 < x < 1 \\ x^2 & 1 \leq x \end{cases}$$

Esercizio 3

- (a) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^3}}{\sqrt[3]{x^2}}$
- (b) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x} + x}{\sqrt[3]{x^2} + \sqrt{x^5}}$
- (c) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^4 + 3}}{2x^2 + 1}$
- (d) $\lim_{x \rightarrow +\infty} \frac{1}{\sqrt{x^5 + 5} + \sqrt{x^3 + 3}}$
- (e) $\lim_{x \rightarrow -\infty} \frac{\sqrt[3]{3x^2 + 4}}{\sqrt[3]{4x^2 + 3}}$

Esercizio 4

- (a) $\lim_{x \rightarrow 7} \frac{\sqrt{7} - \sqrt{x}}{x - 7}$
- (b) $\lim_{x \rightarrow +\infty} \sqrt{x^4 - x^3} - \sqrt{x^4 + x^3}$
- (c) $\lim_{x \rightarrow +\infty} \sqrt{x^4 - x^2} - \sqrt{x^4 + x^2}$
- (d) $\lim_{x \rightarrow +\infty} \sqrt{x^4 - x} - \sqrt{x^4 + x}$
- (e) $\lim_{x \rightarrow +\infty} \frac{\sqrt{x^2 + a^2} - \sqrt{x^2 - b^2}}{\sqrt{x^2 + c^2} - \sqrt{x^2 - d^2}} \quad \text{con } d \neq 0$
- (f) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos^2(x) + 2\text{sen}(x)}}{\text{sen}(x)}$

Esercizio 5

- (a) $\lim_{x \rightarrow 0} \frac{\text{sen}(5x)}{10x}$
- (b) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos(x)}}{\text{tg}^2(x)}$
- (c) $\lim_{x \rightarrow \pi} \frac{\text{sen}(x)}{\pi - x}$
- (d) $\lim_{x \rightarrow 0} \frac{\text{sen}(2x)}{\text{tg}(3x)}$
- (e) $\lim_{x \rightarrow 0} \frac{x(1 - \cos(x))}{\text{sen}(2x) \cdot \text{tg}(3x^2)}$
- (f) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos(x) + x^2}}{\text{sen}^2(x)}$

$$\begin{aligned}
(g) \quad & \lim_{x \rightarrow 1} \frac{\operatorname{tg}(\lg(x))}{\operatorname{sen}(\lg(x))} \\
(h) \quad & \lim_{x \rightarrow 0^+} \frac{\sqrt{1 - \cos(x)}}{x} \\
(i) \quad & \lim_{x \rightarrow 0^-} \frac{\sqrt{1 - \cos(x)}}{x} \\
(l) \quad & \lim_{x \rightarrow 0} \frac{\sqrt{1 + \operatorname{sen}(x)} - \sqrt{1 - \operatorname{sen}(x)}}{\operatorname{sen}(x)} \\
(m) \quad & \lim_{x \rightarrow 0} \frac{\operatorname{sen}(ax)}{b}
\end{aligned}$$

Esercizio 6

$$\begin{aligned}
(a) \quad & \lim_{x \rightarrow +\infty} \frac{\operatorname{sen}(x)}{x} \\
(b) \quad & \lim_{x \rightarrow -\infty} \frac{\cos(x)}{x} \\
(c) \quad & \lim_{x \rightarrow +\infty} x \cdot \operatorname{sen}\left(\frac{1}{x}\right) \\
(d) \quad & \lim_{x \rightarrow +\infty} x \cdot \operatorname{sen}(x)
\end{aligned}$$

Esercizio 7

$$\begin{aligned}
(a) \quad & \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x}\right)^{2x} \\
(b) \quad & \lim_{x \rightarrow +\infty} \left(1 + \frac{a}{x}\right)^{bx} \quad \text{con } a > 0, b > 0 \\
(c) \quad & \lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x^2}\right)^{2x} \\
(d) \quad & \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{\operatorname{sen}(x)}} \\
(e) \quad & \lim_{x \rightarrow 1} x^{\frac{1}{\ln(x)}} \\
(f) \quad & \lim_{x \rightarrow 0} (\cos(x))^{\frac{1}{x^2}} \\
(g) \quad & \lim_{x \rightarrow +\infty} \left(\frac{x}{1 + x}\right)^x \\
(h) \quad & \lim_{x \rightarrow 0} (1 + \operatorname{tg}(x))^{\frac{x}{1 - \cos(x)}} \\
(i) \quad & \lim_{x \rightarrow 0} \frac{\ln(1 + 2x)}{x}
\end{aligned}$$

$$(l) \quad \lim_{x \rightarrow 0} \frac{\ln(1+ax)}{bx} \quad \text{con } a \neq 0, b \neq 0$$

$$(m) \quad \lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{x^2}$$

$$(n) \quad \lim_{x \rightarrow 0} \frac{e^{x^2} - \cos(x)}{x^2}$$

$$(o) \quad \lim_{x \rightarrow +\infty} (\ln(x+1) - \ln(x-1))$$

$$(p) \quad \lim_{x \rightarrow +\infty} x \cdot \ln\left(1 + \frac{a}{x}\right)$$

$$(q) \quad \lim_{x \rightarrow +\infty} x (\ln(x+a) - \ln(x))$$

$$(r) \quad \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{2x}$$

$$(s) \quad \lim_{x \rightarrow 1} \frac{e^x - e}{x - 1}$$

$$(t) \quad \lim_{x \rightarrow 0} \frac{e^x - 1}{\sin(x)}$$

$$(u) \quad \lim_{x \rightarrow 0} \frac{\sin(x) (e^{\tan(x)} - 1)}{\cos(x) - 1}$$

SOLUZIONI

Esercizio 1 a) $\{0\}$ b) $\{\pi\}$ c) $\{-1, 0\}$ d) $\{\frac{\pi}{2}\}$ e) $\{\frac{-\pi}{4}\}$ f) $\{\}$

Esercizio 2 a) $a = 1$ b) $a = 1$ c) $a = 0, b = 1$ d) $a = 1, b = 0$

Esercizio 3 a) $+\infty$ b) 0 c) $\frac{1}{2}$ d) 0 e) $\sqrt[3]{\frac{3}{4}}$

Esercizio 4 a) $-\frac{1}{2\sqrt{7}}$ b) $-\infty$ c) -1 d) 0 e) $\frac{a^2+b^2}{c^2+d^2}$ f) -1

Esercizio 5 a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) 1 d) $\frac{2}{3}$ e) $\frac{1}{12}$ f) $-\frac{1}{4}$ g) 1 h) $\frac{\sqrt{2}}{2}$ i) $-\frac{\sqrt{2}}{2}$ l) 1 m) 0

Esercizio 6 a) 0 b) 0 c) 1 d) Non esiste.

Esercizio 7 a) e^2 b) e^{ab} c) 1 d) e e) e f) $\frac{1}{\sqrt{e}}$ g) $\frac{1}{e}$ h) e^2 i) 2 l) $\frac{a}{b}$ m) 1 n) $\frac{3}{2}$ o) 0 p) a q) a r) $\frac{3}{2}$ s) e t) 1 u) -2