

Esercizi 4.

29/10/2012

Esercizio 1

Determinare la misura in radianti dei seguenti angoli espressi in gradi.

$$(a) \quad 240^\circ; 135^\circ; 330^\circ.$$

Esercizio 2

Calcolare i seguenti valori:

$$(a) \quad \cos\left(\frac{4}{3}\pi\right); \sin\left(\frac{3}{4}\pi\right); \tan\left(\frac{11}{6}\pi\right).$$

Esercizio 3

Verificare le seguenti identità.

$$(a) \quad 1 + \operatorname{tg}^2(\alpha) = \frac{1}{\cos^2(\alpha)}$$

$$(b) \quad 1 + \operatorname{ctg}^2(\alpha) = \frac{1}{\sin^2(\alpha)}$$

$$(c) \quad \sin^4(\alpha) - \cos^4(\alpha) = \sin^2(\alpha) - \cos^2(\alpha)$$

Esercizio 4

Risolvere le seguenti equazioni:

$$(a) \quad \sin(2x) = 1$$

$$(b) \quad 1 - \sqrt{2} \cdot \cos(x) = 0$$

$$(c) \quad \cos(\lambda x) = 1 \quad \text{supporre } \lambda > 0$$

$$(d) \quad \sin(3x) = 3$$

$$(e) \quad \sin^2(x) = \frac{1}{2}$$

$$(f) \quad \cos^2(x) - 3\cos(x) - 4 = 0$$

$$(g) \quad 2\sin^2(x) - \sin(x) = 1$$

$$(h) \quad \cos(x) + 1 = 2\sin^2(x)$$

Esercizio 5

Risolvere le seguenti disequazioni

$$(a) \quad 2\cos(x) < \sqrt{3}$$

$$(b) \quad \sin^2(x) < \frac{3}{4}$$

$$(c) \quad |\cos(x)|^2 > \frac{3}{4}$$

Esercizio 6

Sul piano cartesiano sia $A = (6, 0)$, sia $B = (3, 4)$, sia $\theta = \widehat{AOB}$ e sia P un punto del primo quadrante tale che $\overline{OP} = 6$ e $\widehat{AOP} = \frac{\pi}{6}$.

- (a) Calcolare la lunghezza di $\overline{OB}$
- (b) Calcolare $\sin(\theta)$
- (c) Calcolare $\cos(\theta)$
- (c) Calcolare le coordinate del punto P

Esercizio 7

Calcolare i seguenti limiti.

$$(a) \quad \lim_{x \rightarrow 3} x^2 - 2x - 1$$

$$(b) \quad \lim_{x \rightarrow 0} \frac{\sqrt{x+5}}{1+x}$$

$$(c) \quad \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{8}\sin(x) - 1}{2x}$$

$$(d) \quad \lim_{x \rightarrow -\frac{\pi}{3}} \frac{\cos(x) + 1}{\sqrt{3} + \sin(x)}$$

$$(e) \quad \lim_{x \rightarrow 0^+} \frac{1}{\sin(x)}$$

$$(f) \quad \lim_{x \rightarrow 0^-} \frac{1}{\sin(x)}$$

$$(g) \quad \lim_{x \rightarrow +\infty} \sin\left(\frac{1}{\log(x)}\right)$$

Esercizio 8

Indicare (se vi sono) i punti di discontinuità delle seguenti funzioni.

$$(a) \quad f(x) = \begin{cases} 0 & x \leq 0 \\ 1+x & x > 0 \end{cases}$$

$$(b) \quad f(x) = \begin{cases} 0 & x \leq 0 \\ \sin(x) & 0 < x \leq \pi \\ \cos(x) & \pi < x \end{cases}$$

$$(c) \quad f(x) = \begin{cases} e^{-x} & x < -1 \\ x+1 & -1 \leq x \leq 0 \\ x^2 & 0 < x \leq 1 \\ x & 1 < x \end{cases}$$

$$(d) \quad f(x) = \begin{cases} \sin(x) & x \leq 0 \\ \operatorname{tg}(x) & 0 < x < \frac{\pi}{2} \\ \cos(x) & \frac{\pi}{2} \leq x \end{cases}$$

$$(e) \quad f(x) = \begin{cases} \cos(x) & x \leq -\frac{\pi}{4} \\ \sin(x) & -\frac{\pi}{4} < x < \frac{\pi}{4} \\ \cos(x) & \frac{\pi}{4} \leq x \end{cases}$$

$$(f) \quad f(x) = \begin{cases} \frac{x^2-4}{x-2} & x \neq 2 \\ 4 & x = 2 \end{cases}$$

Esercizio 9

Per quali valori di a e b le seguenti funzioni sono continue.

$$(a) \quad f(x) = \begin{cases} a & x \leq 0 \\ \cos(x) & x > 0 \end{cases}$$

$$(b) \quad f(x) = \begin{cases} \frac{\sin(x)}{x} & x \neq 0 \\ a & x = 0 \end{cases}$$

$$(c) \quad f(x) = \begin{cases} e^{-x} & x < 0 \\ a + 1 & x = 0 \\ be^x & x > 0 \end{cases}$$

$$(d) \quad f(x) = \begin{cases} \cos(x) & x \leq 0 \\ a + bx & 0 < x < 1 \\ x^2 & 1 \leq x \end{cases}$$

Esercizio 10

$$(a) \quad \lim_{x \rightarrow 0} \frac{\sin(5x)}{10x}$$

$$(b) \quad \lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos(x)}}{\operatorname{tg}^2(x)}$$

$$(c) \quad \lim_{x \rightarrow \pi} \frac{\sin(x)}{\pi - x}$$

$$(d) \quad \lim_{x \rightarrow 0} \frac{\sin(2x)}{\operatorname{tg}(3x)}$$

$$(e) \quad \lim_{x \rightarrow 0} \frac{x(1 - \cos(x))}{\sin(2x) \cdot \operatorname{tg}(3x^2)}$$

$$(f) \quad \lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos(x) + x^2}}{\sin^2(x)}$$

$$(g) \quad \lim_{x \rightarrow 1} \frac{\operatorname{tg}(\lg(x))}{\sin(\lg(x))}$$

$$(h) \quad \lim_{x \rightarrow 0^+} \frac{\sqrt{1 - \cos(x)}}{x}$$

$$(i) \quad \lim_{x \rightarrow 0^-} \frac{\sqrt{1 - \cos(x)}}{x}$$

$$(l) \quad \lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin(x)} - \sqrt{1 - \sin(x)}}{\sin(x)}$$

$$(m) \quad \lim_{x \rightarrow 0} \frac{\sin(ax)}{b}$$

Esercizio 11

$$(a) \quad \lim_{x \rightarrow +\infty} \frac{\sin(x)}{x}$$

$$(b) \quad \lim_{x \rightarrow -\infty} \frac{\cos(x)}{x}$$

$$(c) \quad \lim_{x \rightarrow +\infty} x \cdot \sin\left(\frac{1}{x}\right)$$

$$(d) \quad \lim_{x \rightarrow +\infty} x \cdot \sin(x)$$

SOLUZIONI

Esercizio 1

(a) $\frac{4}{3}\pi; \frac{3}{4}\pi; \frac{11}{6}\pi.$

Esercizio 2

(a) $-\frac{1}{2}; \frac{\sqrt{2}}{2}; -\frac{\sqrt{3}}{3}.$

Esercizio 4

- a) $\{\frac{\pi}{4} + k\pi : k \in \mathbb{Z}\}$
 b) $\{\frac{\pi}{4} + 2k\pi : k \in \mathbb{Z}\} \cup \{-\frac{\pi}{4} + 2k\pi : k \in \mathbb{Z}\}$
 c) $\{\frac{2k\pi}{\lambda} : k \in \mathbb{Z}\}$
 d) $\emptyset$
 e) $\{\frac{\pi}{4} + \frac{k\pi}{2} : k \in \mathbb{Z}\}$
 f) $\{\pi + 2k\pi : k \in \mathbb{Z}\}$
 g) $\{\frac{\pi}{2} + 2k\pi : k \in \mathbb{Z}\} \cup \{\frac{7\pi}{6} + 2k\pi : k \in \mathbb{Z}\} \cup \{\frac{11\pi}{6} + 2k\pi : k \in \mathbb{Z}\}$
 h) $\{\pi + 2k\pi : k \in \mathbb{Z}\} \cup \{\frac{\pi}{3} + 2k\pi : k \in \mathbb{Z}\} \cup \{\frac{5\pi}{3} + 2k\pi : k \in \mathbb{Z}\}$

Esercizio 5

- a) $\cup_{k \in \mathbb{Z}} (\frac{\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi)$
 b) $\cup_{k \in \mathbb{Z}} (-\frac{\pi}{3} + k\pi, \frac{\pi}{3} + k\pi)$
 c) $\cup_{k \in \mathbb{Z}} (-\frac{\pi}{6} + k\pi, \frac{\pi}{6} + k\pi)$

Esercizio 6 a) 5 b) $\frac{4}{5}$ c) $\frac{3}{5}$ d) $P = (3\sqrt{3}, 3)$

Esercizio 7 a) 2 b) $\sqrt{5}$ c) $\frac{2}{\pi}$ d) $\sqrt{3}$ e) $+\infty$ f) $-\infty$ g) 0

Esercizio 8 a) $\{0\}$ b) $\{\pi\}$ c) $\{-1, 0\}$ d) $\{\frac{\pi}{2}\}$ e) $\{\frac{-\pi}{4}\}$ f) $\{\}$

Esercizio 9 a) $a = 1$ b) $a = 1$ c) $a = 0, b = 1$ d) $a = 1, b = 0$

Esercizio 10 a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) 1 d) $\frac{2}{3}$ e) $\frac{1}{12}$ f) $-\frac{1}{4}$ g) 1 h) $\frac{\sqrt{2}}{2}$ i) $-\frac{\sqrt{2}}{2}$ l) 1
 m) 0

Esercizio 11 a) 0 b) 0 c) 1 d) Non esiste.