

Esercizi 8.

10/12/2012

Esercizio 1

Limiti con la regola di de l'Hôpital.

$$(a) \quad \lim_{x \rightarrow 0} \frac{(3-x)e^x - x - 3}{x^3}$$

$$(b) \quad \lim_{x \rightarrow 0} \frac{\log(1+x) - \sin(x)}{x^2}$$

$$(c) \quad \lim_{x \rightarrow 0^+} \frac{x - \sin(x)}{(x \cdot \sin(x))^{\frac{3}{2}}}$$

$$(d) \quad \lim_{x \rightarrow (\frac{1}{2})^-} \frac{\log(1-2x)}{\tan(\pi x)}$$

$$(e) \quad \lim_{x \rightarrow 1^-} \log(x) \cdot \log(1-x)$$

$$(f) \quad \lim_{x \rightarrow 0^+} \frac{e^x - \cos(x) + 1}{x^2 + 1}$$

Esercizio 2

Calcolare le seguenti somme.

$$(a) \quad \sum_{i=1}^{200} i$$

$$(b) \quad \sum_{i=100}^{200} i$$

$$(c) \quad \sum_{i=0}^{10} \left(\frac{1}{2}\right)^i$$

$$(d) \quad \sum_{i=0}^n \left(\frac{1}{3}\right)^i$$

$$(e) \quad \sum_{i=0}^{\infty} \left(\frac{1}{3}\right)^i$$

Esercizio 3

Calcolare i seguenti integrali indefiniti.

$$(a) \quad \int 2x^7 + x^4 \, dx$$

$$(b) \quad \int 3\text{sen}(x) + 4\cos(x) \, dx$$

$$(c) \quad \int \text{sen}(3x) + \cos(4x) \, dx$$

$$(d) \quad \int 5e^x + \frac{1}{6x} \, dx$$

$$(e) \quad \int e^{5x} + \frac{1}{x+1} \, dx$$

Esercizio 4

Risolvere per sostituzione i seguenti integrali.

$$(a) \quad \int x^2(1 + 3x^3)^4 \, dx$$

$$(b) \quad \int \frac{x^2}{(x^3 + 1)^2} \, dx$$

$$(c) \quad \int \frac{e^x}{e^x + 1} \, dx$$

$$(d) \quad \int \frac{1}{x + \sqrt{x}} \, dx$$

$$(e) \quad \int \frac{\cos(x)}{\text{sen}^4(x)} \, dx$$

Sostituzioni: (a) $t = 1 + 3x^3$. (b) $t = x^3 + 1$. (c) $t = e^x$. (e) $t = \text{sen}(x)$

Esercizio 5

Risolvere per parti i seguenti integrali.

$$(a) \quad \int x \cdot e^{-x} \, dx$$

$$(b) \quad \int 3x \cdot \text{sen}(x) \, dx$$

$$(c) \quad \int x \cdot \cos(4x) \, dx$$

$$(d) \quad \int x^2 \cdot \log(x) \, dx$$

$$(e) \quad \int x^2 \cdot e^{2x} dx$$

Esercizio 6

Risolvere i seguenti integrali.

$$(a) \quad \int_{-1}^2 2x^3 + 4x dx$$

$$(b) \quad \int_1^e x^3 \log(x) dx$$

$$(c) \quad \int_0^1 x^3 e^{x^4} dx$$

$$(d) \quad \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\text{sen}(x) + 7\cos(x) dx$$

$$(e) \quad \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2(x)} dx$$

SOLUZIONI

Esercizio 1 a) $+\infty$ b) $-\frac{1}{2}$ c) $\frac{1}{6}$ d) 0 e) 0 f) 1

Esercizio 2 a) 20100 b) 15150 c) $2 - \frac{1}{1024}$ d) $\frac{3}{2} \left(1 - \left(\frac{1}{3}\right)^{n+1}\right)$ e) $\frac{3}{2}$

Esercizio 3 a) $\frac{x^8}{4} + \frac{x^5}{5} + c$ b) $-3\cos(x) + 4\text{sen}(x) + c$ c) $-\frac{\cos(3x)}{3} + \frac{\text{sen}(4x)}{4} + c$
d) $5e^x + \frac{1}{6}\log(|x|) + c$ e) $\frac{e^{5x}}{5} + \log(|x+1|) + c$

Esercizio 4 a) $\frac{(1+3x^3)^5}{45} + c$ b) $-\frac{1}{3(x^3+1)} + c$ c) $\log(e^x+1) + c$ d) $2\log(\sqrt{x}+1) + c$
e) $-\frac{1}{3\text{sen}^3(x)} + c$

Esercizio 5 a) $-xe^{-x} - e^{-x} + c$ b) $-3x \cdot \cos(x) + 3 \cdot \text{sen}(x) + c$ c) $\frac{\cos(4x)}{16} + \frac{x\text{sen}(4x)}{4} + c$ d) $\frac{x^3}{3}(\log(x) - \frac{1}{3}) + c$ e) $e^{2x}(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}) + c$

Esercizio 6 a) 13.5 b) $\frac{3e^4+1}{16}$ c) $\frac{e-1}{4}$ d) 14 e) 1