Random Graphs and Networks

Prof. Alessandra Bianchi

Timetable: 24 hrs. First lecture on January 25, 2022, 11:00 (date already fixed, see the Calendar of activities at https://dottorato.math.unipd.it/calendar).

Location: The course will be held in the Math. Department of Padova according with the following calendar

Jan 25, 26 - Feb 1, 2, 3, 8, 17 in Room 2BC60
Jan 27 - Feb 10 in Room 1BC50
Feb 9, 15, 16 in Room 2AB45

and online by zoom - Meeting ID: 812 1156 9941 / Passcode: 294788
The link to access the meeting is also available at the webpage
https://www.math.unipd.it/~bianchi/PhD21.html where all the material of the course will be collected.

Course requirements: Basic knowledge of probability theory: discrete random variables, finite and countable probability spaces, convergence theorems (law of large number, central limit theorem).

Examination and grading: Seminar.

SSD: MAT/06 Probability and Mathematical Statistics

Course contents: Complex networks are attracting in recent years an increasing attention of the scientific community, due to the wide range of real-world situations in which they arise. Examples of networks emerge from the internet, from social relations and collaborations, from electrical power grids, or from biological and ecological environments. Important common features of all these examples appear from their large-scale behavior, where their share similar properties such as the "small words" and the "scale free" phenomena. Random graphs are the mathematical models that allow to analyze these large scale features; roughly, random graphs can be described as random variables taking values on a set of graphs, hence well suited to capture both probabilistic and combinatorial aspects of the real-world networks listed above.
The course will focus on different classes of random graphs. We will start from the definition of the Erdos-Rényi random graph, one of simplest model one could think of. Despite its simplicity, this model presents relevant and unforeseen large-scale features that will be discussed along the course, including an interesting phase transition related to presence of a giant connected component. Keeping in mind the properties of real networks, we will then introduce and discuss three different families of random graphs: The Inhomogeneous Random Graph, the Configuration Model and the Preferential Attachment Model. For these random graphs we will prove precise asymptotic results for the degree distributions, and discuss their scale free and small world behavior.

Lectures (tentative schedule)

1. Basic setting: graphs, trees, random graph setting, and main properties of the real-world networks.
2. Erdos-Renyi (ER) random graphs: Uniform and Binomial model; monotonicity and thresholds.
3. ER random graphs structure: trees containment, Poisson paradigm, largest component, connectivity.
4. Branching Processes: survival probability; total progeny; random walk perspective, duality principle.
5. Exploration process; largest component in subcritical regime.
6. Emergence of a giant component in ER- random graphs: Phase transition and behavior at criticality.
7. Inhomogeneous random graphs (IRG): degree sequence and scale-free property.
8. Configuration Model (CM): construction and simplicity probability. Uniform random graphs.
9. Multi-type branching process; local convergence of random graphs.
10. Phase transition and small world phenomenon in the IRG and in the CM.
11. Preferential Attachment Model (PAM): construction, scale free and small world properties.
12. Perspectives: stochastic models on random graphs.

References

[1] R. van der Hofstad. *Random graphs and complex networks. Vol. 1*. Cambridge Series in Statistical and Probabilistic Mathematics, [43]. Cambridge University Press, Cambridge, 2017.
Available on the author webpage: https://www.win.tue.nl/~rhofstad/NotesRGCN.pdf

[2] R. van der Hofstad. *Random graphs and complex networks. Vol. 2*. To appear in Cambridge Series in Statistical and Probabilistic Mathematics. A preliminary version in pdf is available on the author webpage https://www.win.tue.nl/~rhofstad/NotesRGCNII.pdf

[3] A. Frieze, M. Karoński. *Introduction to random graphs.* Cambridge University Press, Cambridge, 2016.
Available on the author webpage: https://www.math.cmu.edu/~af1p/BOOK.pdf

[4] R. van der Hofstad. *Stochastic processes on random graphs*. Lecture notes for the 47th Summer School in Probability Saint-Flour 2017.
Available on the author webpage: https://www.win.tue.nl/~rhofstad/SaintFlour_SPoRG.pdf