Session 25 – ARITHMETIC ALGEBRAIC GEOMETRY

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Density Functions and Symmetry Groups of Families of Elliptic Curves

ABSTRACT: Based on theory from the function field case, we expect every natural family of L-functions has an associated symmetry group controlling the distribution of zeros. For elliptic curves, we expect $SO(even)$ if every curve is even, $SO(odd)$ if all are odd, and O if the signs are equi-distributed. By studying the n-level density (defined by summing a test function over the zeros), we obtain a statistic which is different for each of the candidate groups. Previous investigations for families of elliptic curves have only succeeded in evaluating the 1-level density for functions supported in $(-1, 1)$, where the three candidates are indistinguishable. In this talk we calculate the 2-level density in a restricted range, often for families with forced rank over $\mathbb{Q}(t)$. We show the three orthogonal candidates have different 2-level densities for test functions supported in an arbitrarily small neighborhood of the origin. Assuming standard conjectures, we observe the expected orthogonal group.

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