Addendum to: Spherical conjugacy classes and involutions in the Weyl group, Math. Z. 260(1) 1–23 (2008)

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For a dominant weight λ, let $V(\lambda)$ be the corresponding Weyl module and let $L(\lambda)$ be its irreducible quotient. Let $k[G/H]$ denote the ring of regular functions on a homogeneous space and $k[\mathcal{O}]$ denote the ring of regular functions on a conjugacy class $\mathcal{O}$. By [2, Theorem 1.2], erroneously cited in the paper, if a homogeneous G-space G/H is spherical then $\dim\mathrm{Hom}(V(\lambda), k[G/H]) \leq 1$. In particular, since $L(\lambda)$ is a quotient of $V(\lambda)$, we have an injection $\mathrm{Hom}(L(\lambda), k[G/H]) \to \mathrm{Hom}(V(\lambda), k[G/H])$, so $\dim \mathrm{Hom}(L(\lambda), k[G/H]) \leq 1$ and the multiplicity of $L(\lambda)$ in the socle of the G-module $k[G/H]$ is at most 1. For this reason we called $k[G/H]$ *multiplicity-free*. The reader should be warned that the same term is used in [1] with a different meaning. We add some details to the statement and the proof of Theorem 4.17 when the characteristic of k is positive.

Theorem 0.1 *Let $\mathcal{O}, v_0, w, \Pi$ be as in Theorem 4.4. If $\mathrm{Hom}(V(\lambda), k[\mathcal{O}]) \neq 0$ then $-w_0\lambda = \lambda$ and $\lambda \in P^+ \cap Q \cap \mathrm{Ker}(1+w)$. In particular, this happens for the weights for which $L(\lambda)$ occurs in the socle of $k[\mathcal{O}]$.*

Proof. If $\mathrm{Hom}(V(\lambda), k[\mathcal{O}]) \neq 0$ there exists $0 \neq f \in k[\mathcal{O}]$ such that $t.f(x) = f(t^{-1}.x) = \lambda(t)f(x)$ for all x in an open subset of $\mathcal{O}$. In particular, for $x \in \dot{w}U$ and every $t \in (T^w)^\circ$ we have $\lambda(t) = 1$. We proceed as in Theorem 4.7. $\square$

References

[1] J. BRUNDAN, *Multiplicity-free subgroups of reductive algebraic groups,* J. Algebra 188(1) 310–330 (1997).

[2] J. BRUNDAN, *Dense orbits and double cosets,* In: Algebraic groups and their representations (Cambridge, 1997), 259–274, NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., 517, Kluwer Acad. Publ., Dordrecht (1998).