Appendix – Proof of the Statement in Section 4.1

From the first assumption, we can say that, if the i-th node successes in transmitting, then $(i-1)$-th, $(i-2)$-th, $(i+1)$-th and $(i+2)$-th cannot.

Let us name the following events: *(i)* D_n be the event of delivering a packet in a chain of n links and *(ii)* S_n^i be the event of delivering at the i-th attempt.

Let us name $T_{i,n}$ as the probabilistic event of delivering a packet in a network of n links (i.e., $n+1$ nodes) after i retransmissions [8].

For all n the probability of delivering after one attempt is the same as the probability of deliver a packet: $P(T_{1,n}) = P(D_n)$. Conversely, probability $P(T_{2,n})$ is equal to the probability of not delivering at the first $P(\neg S_n^1)$ and of delivering at the second attempt $P(S_n^2)$:

$$P(T_{2,n}) = P(S_n^2 \cap \neg S_n^1) = P(S_n^2) \cdot P(\neg S_n^1 | S_n^2) \quad (3)$$

Since, for all i, events S_n^i are independent and $P(S_n^i) = P(D_n)$, Equation 3 becomes:

$$P(T_{2,n}) = P(S_n^2) \cdot P(\neg S_n^1) = P(D_n) \cdot (1 - P(D_n))$$

In general, the probability of delivering a packet to the destination node after i retransmissions is:

$$\begin{aligned} P(T_{i,n}) &= P(S_n^i) \cdot P(\neg S_n^{(i-1)}) \cdot \ldots \cdot P(\neg S_n^1) = \\ &= P(D_n) \cdot (1 - P(D_n))^{i-1} \end{aligned} \quad (4)$$

We can compute the average number of retransmissions, according to Equation 4 as follows:

$$\begin{aligned} T_n &= \sum_{i=1}^{\infty} P(T_{i,n}) = \\ &= \sum_{i=1}^{\infty} P(D_n) \cdot (1 - P(D_n))^{i-1} = \tfrac{1}{P(D_n)} \end{aligned} \quad (5)$$

In a laboratory, all nodes are in the same radio range. Therefore, independently on the nodes number,

$$P(D_n^{lab}) = 1/n \quad (6)$$

. On the field, we have to distinguish on the basis of the number of links. Up to 2 links (i.e., 3 nodes), all nodes interfere and, hence, just one node out of 2 or 3 can deliver a packet in a time slot. So, $P(D_1^{field}) = 1$ and $P(D_2^{field}) = 1/2$. For links $n = 3, 4, 5$, two nodes success: $P(D_n^{field}) = 2/n$. For links $n = 6, 7, 8$, there are 3 nodes delivering: $P(D_n^{field}) = 3/n$. Hence, in general we can state:

$$P(D_n^{field}) = \frac{\lfloor \frac{n}{3} \rfloor + 1}{n}. \quad (7)$$

By applying Equations 6 and 7 to Equation 5, we derive the number of retransmission needed for delivering a packet :

$$\begin{aligned} T^{field}(n) &= \tfrac{n}{\lfloor \frac{n}{3} \rfloor + 1} \\ T^{lab}(n) &= n. \end{aligned} \quad (8)$$

[8] Please note this is different with respect to S_n^i, since $T_{i,n}$ implies deliver did not success up to the $i-1$-th attempt

Fixing the number of packets to be delivered, we can define a function f that expresses the throughput in function of the number of sent packets. If we have a chain of n links and we want to deliver a single packet from the first to the last node in the chain, then we have altogether to send the number n of links times the expected value for each link T_n. Therefore:

$$\begin{aligned} Q_{lab}(n) &= f(T^{lab}(n) \cdot n) = f(n^2) \\ Q_{field}(n) &= f(T^{field}(n) \cdot n) = f(\tfrac{n^2}{\lfloor \frac{n}{3} \rfloor + 1}) \end{aligned} \quad (9)$$

From our laboratory experiments described in Section 4.2, as well as from other theoretical results [9]), we can state $f(n^2) = \frac{\alpha}{n^\beta}$. By considering it and Equations 9, the following holds:

$$\frac{Q_{lab}(n)}{f(n^2)} = \frac{Q_{field}(n)}{f(\frac{n^2}{\lfloor \frac{n}{3} \rfloor + 1})} \Rightarrow Q_{field}(n) = Q_{lab}(n) \cdot \left(\lfloor \frac{n}{3} \rfloor + 1\right)^{\frac{\beta}{2}}. \quad (10)$$