# Lab exercises

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Solve the following problems

1. Take the function $f(x) = x^2 - \log(x^2+2)$, of which we want to compute its zeros. Then, consider the iteration functions $g_{1,2}(x) = \pm\sqrt{\log(x^2+2)}$, which converge to the positive and the negative root, respectively (see below the graph of $f$ and $g_1'$)

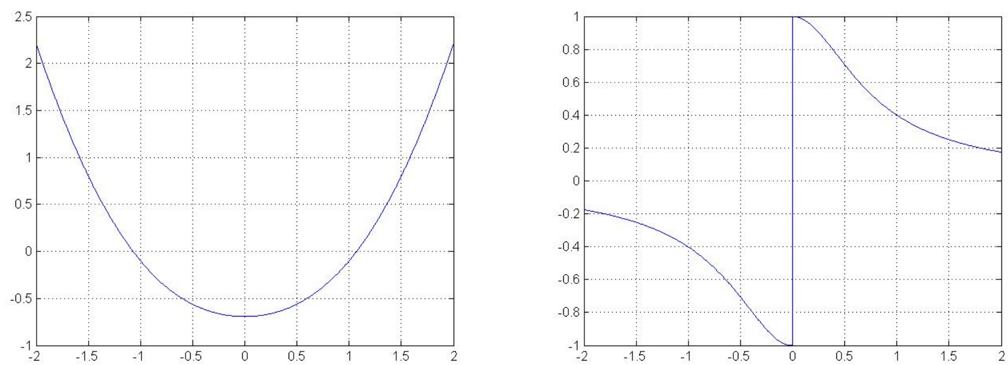


Figure 1: The plot of $f$ and $g'$ of Exercise **??**

   - Plot relative error showing a linear convergence of the method
   - By using the *Aitken acceleration method*, show that the sequence $x_{k+1} = g(x_k)$ can be accelerated at least with order of convergence 2.

     For helping the implementation of Aitken's method, the following steps should be done

     (a) Choose $x = 0$, the tolerance $\epsilon$ and maximum number of iterations `max_it`.

     (b) Compute $x_1 = g(x_0)$ and $x_2 = g(x_1)$.

     (c) Then set the first element of the Aitken's sequence $y = x_0 - \dfrac{(x_1 - x_0)^2}{x_2 - 2x_1 + x_0}$.

     (d) Set the new $x_0 = y$ and $x_1, x_2$ as before.

     (e) Continue until $|y - x_2| \geq \epsilon$ and the iterations are less than `max_it`.

2. Take the function $f(x) = \tan(x) - \log(x^2+2)$, $x \in [0,1]$, which has a zero $\alpha \approx 0.76$. Consider the fixed point iteration with $g(x) = \arctan(\log(x^2+2))$. Compare the behavior of this iterative scheme and the Aitken's one for the computation of $\alpha$

3. Implement the *Steffensen's method* for solving $f(x) = (x-1)e^x = 0$ (which has the unique zero in $x = 1$). Compare it with the fixed point iteration schemes

$$g_1(x) = \log(x\, e^x),\ g_2(x) = \frac{e^x + x}{e^x + 1},\ g_3(x) = \frac{x^2 - x + 1}{x}$$

by checking the convergence evaluating their first derivatives at the zero. Take as initial value, tolerance and max number of iteration the values $x_0 = 2,\ tol = 10^{-10},\ nmax = 100$, respectively.

Just to remind, the sequence generated by Steffensen is the following

$$x_{k+1} = x_k - \frac{(f(x_k))^2}{f(x_k + f(x_k)) - f(x_k)}\,, \quad k \geq 0.$$

Time: **2 hours**.