# Lab exercises

*Prof. S. De Marchi*
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## 1 Solution of linear systems with iterative methods

1. Write an M-function

   `function [x iter err] = Jacobi(A,b,tol,maxit)`

   Then, test it for the solution of the linear system $Ax = b$ with `A = toeplitz([4 1 0 0 0 0])` and $b$ chosen so that the exact solution of the system is $x = [2, 2, 2, 2, 2, 2]^T$. As initial vector, take `x0=zeros(6,1)`. The output parameters are: `x` the solution vector, `iter` the number of iterations and `err` the vector of relative errors among iterations. Make also a plot, using `semilogy`, of the error.

   Do the same with the Gauss-Seidel iteration.

2. Choose $n$ and build the tridiagonal matrix

   `A=diag(ones(n,1)*10)+diag(ones(n-1,1)*3,+1)+diag(ones(n-1,1)*3,-1)`

   and the vector `b=(1:n)'`.

   (a) Jacobi and Gauss-Seidel iterations converges for the solution of $Ax = b$. Why? Then, consider the Jacobi matrix $J$ (which has $\|J\| < 1$). Given the tolerance $\epsilon = 1.e - 9$, by using
   $$\frac{\|P\|^k}{1 - \|P\|}\|x^1 - x^0\| < \epsilon$$
   compute a priori the minimum number $k^*$ of iterations necessary to solve the linear system $Ax = b$. Use as $x^0 = \mathbf{0}$, `kmax=50` and $\| \cdot \|_2$.

   (b) Do the same for the Gauss-Seidel iteration.

   (c) Check the consistency of the results by using the Jacobi and Gauss-Seidel iterations.

3. Construct the matrix `A=pentadiag(-1,-1,`$\alpha$`, -1,-1)` with $n = 10$, $\alpha \in [0.5, 1.5]$. Using the decomposition $A = M+D+N$ with `D=diag(`$[\alpha-1, \ldots, \alpha-1]$`)`, `M=pentadiag(-1,-1,1,0, 0,0)` and $N = A - D - M$.

   (a) For which $\alpha^*$ the iterative method $(M + N)x^{(k+1)} = -Dx^{(k)} + q$ results converging faster?

   (b) Letting `b=1:n` , compute the solution of $Ax = b$ starting from the initial solution $x^{(0)}$=`[ ones(m,1); zeros(n-m,1)]`, with $m < n$. Use $tol = 1.e - 6$.

4. Consider the matrix `A=diag(1:n)`, for some $n$. Study the iteration
   $$x^{(k+1)} = (I - \theta A)x^{(k)} + \theta b, \quad k \geq 0\,.$$

(a) For which $\theta \in [0, 2/3]$ the method converges? (*Hint: plot* $\rho(\theta) = \rho(I - \theta A)$*).*

(b) Let $\theta_0 = \min \rho(\theta), 0 \leq \theta \leq 2/3$. Let $b$ such that `x=ones(n,1)` and `x0=zeros(n,1)`. Solve the linear system with the iteration above with for $\theta_0$ with $tol = 1.e - 6$ and `max_it=100`.

Time: **2 hours**.