Lab exercises

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1. Write the M-function that implements the method `SOR`. Apply it to the matrix
   A_α=`pentadiag(-1,-1,`α`,-1,-1)` with $n = 10, \alpha = 3/4$
   of the exercise 3. of the Lab of May 18th, 2015.

2. Take the tridiagonal matrix

$$A(\alpha) = \begin{bmatrix} \alpha & 1 & 0 & \cdots & 0 \\ -1 & \alpha & 1 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & & & -1 & \alpha \end{bmatrix}$$

   The right hand side $b(\alpha)$ be constructed as

$$b_1 = b_n = \alpha + 1,\ b_i = \alpha + 2,\ i = 2, \ldots, n-1\,.$$

   (a) Construct the iterations matrices P_J and P_{GS} and determine if they are convergent.

   (b) Solve the linear system $A(\alpha)x = b(\alpha)$ starting from $x^{(0)} = [1/n, 2/n, \ldots, 1]^T$ with $tol = 1.e-6$ and for $\alpha = 2, 4$ and $n = 8, 16$.

   (c) For which values of α can we determine directely the value of ω^*? For that optimal value, solve the system with the SOR.

3. Consider the random values `x=rand(20,1)` and stretch them to $[0, 5]$. Call this vector again `x`. The corresponding values along y are defined as follows

$$y_i = x_i^2,\ \ for\ i\ odd \tag{1}$$
$$y_i = x_i^2 + 0.5,\ \ for\ i\ even. \tag{2}$$

   Find the polynomial of degree 2 that approximates the couples $(x_i, y_i),\ i = 1, ..., 20$ in the least-squares sense. Try then with a polynomial of degree 3

Time: **2 hours**.