Lab exercises
Degree in mechanical engineering
AY 2014-15

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Solve the following exercises in Matlab.

1. Consider the function $f(x) = \mathtt{e}^x - 4x^2$, whose roots are $\xi_1 \in (-1, 0)$, $\xi_2 \in (0, 1)$ and $\xi_3 \in (4, 4.5)$. Find ξ_2 with `Newton's method` and ξ_3 with the `iterative method` $x_{i+1} = \log(4x_i^2)$.

   This last method converges for every initial point x_0?

2. Consider the function $f(x) = (x-2)^3\mathtt{e}^x, \quad x \in [0, 3]$, that has 2 as a triple root. Using the expression

$$m_k = \frac{x_{k-1} - x_{k-2}}{2x_{k-1} - x_k - x_{k-2}},\ k \geq 2$$

   determine the approximation of the multiplicity of the root. The approximations of $\{x_s\}$ can be computed by Newton's method. Stop when $|m_k - m_{k-1}| \leq 1.e-6$.

3. Take the function

$$f(x) = x^2 - c\,, \quad c \geq 0 \tag{1}$$

   and the following two **iteration functions**:

   (a)
$$g_1(x) = x - \frac{x^2 - c}{2x}\,.$$

   (b)
$$g_2(x) = x - \frac{x^2 - c}{2x} - \frac{\left(x - \frac{x^2-c}{2x}\right)^2 - c}{2x}\,.$$

   Study the convergence of these two iterative methods.

4. Given the polynomial $p(x) = x^3 - 8x^2 + \frac{85}{4}x - \frac{75}{4}, \quad x \in I = [2, 3.5]$.

   (a) Plot $p(x)$ and identify the interval on which lyes the root of biggest modulus, say α_M.

   (b) Find α_M with the iterative method

$$x_{k+1} = y - \frac{f(y)}{f'(x_k)}$$

   with y computed by the `Newton's method`.

Time: **2 hours**.