# Lab exercises

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Solve the following problems

1. Let $f(x) = x^2 - \log(x^2+2)$ be the function of which we want to compute its zeros. Consider the iteration functions $g_{1,2}(x) = \pm\sqrt{\log(x^2+2)}$, which converge to the positive and the negative root, respectively (see below the graphs $g'_{1,2}$)

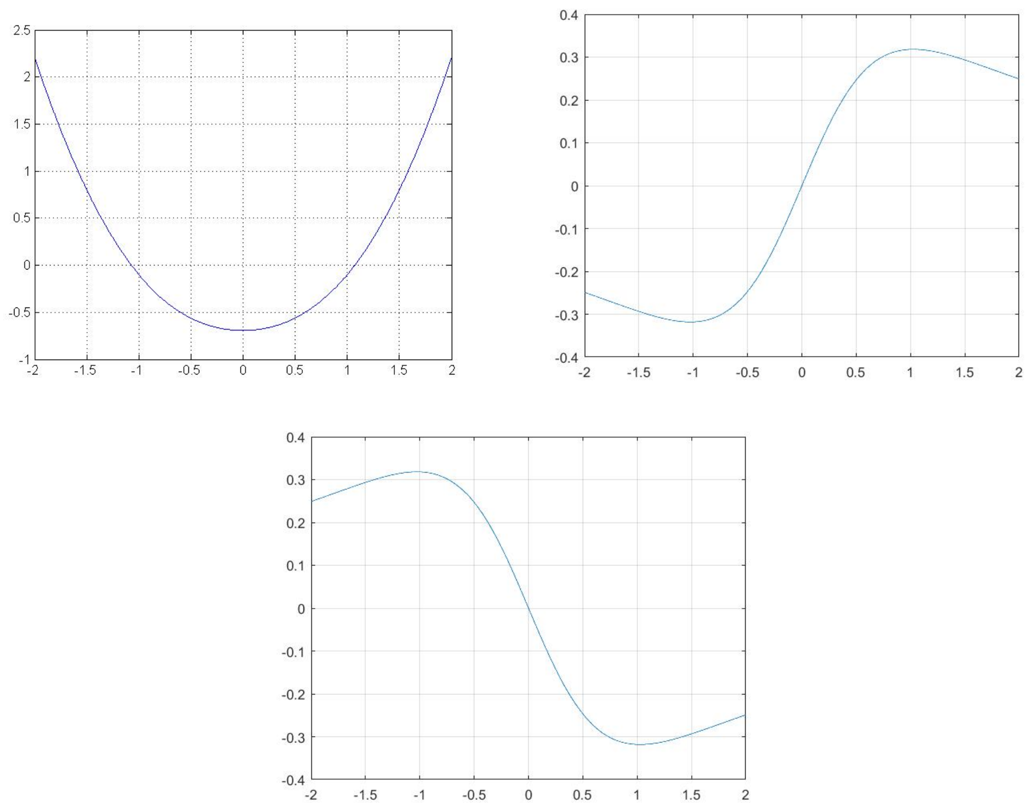


Figure 1: Plots of $f$, $g'_1$ and $g'_2$ of Exercise 1

- Plot relative errors showing a linear convergence of the methods.
- By using the *Aitken acceleration method*, show that the sequence $x_{k+1} = g_{1,2}(x_k)$ can be accelerated at least with order of convergence 2.

  For helping the implementation of Aitken's method, the following steps should be done (*Steffensen's strategy*)

(a) Choose $x = 0$, the tolerance $\epsilon$ and maximum number of iterations `max_it`.

(b) Compute $x_1 = g(x_0)$ and $x_2 = g(x_1)$.

(c) Then set the first element of the Aitken's sequence $y = x_0 - \dfrac{(x_1 - x_0)^2}{x_2 - 2x_1 + x_0}$.

(d) Set the new $x_0 = y$ and $x_1, x_2$ as before.

(e) Continue until $|y - x_2| \geq \epsilon$ and the iterations are less than `max_it`.

2. Take the function $f(x) = \tan(x) - \log(x^2 + 2)$, $x \in [0, 1]$, which has a zero $\alpha \approx 0.76$. Consider the fixed point iteration with $g(x) = \arctan(\log(x^2 + 2))$. Compare the behavior of this iterative scheme and the Aitken's one for the computation of $\alpha$

3. Implement the *Steffensen's method* for solving $f(x) = (x - 1)e^x = 0$ (which has the unique zero in $x = 1$). Just to remind, the sequence generated by Steffensen is the following

$$x_{k+1} = x_k - \frac{(f(x_k))^2}{f(x_k + f(x_k)) - f(x_k)}, \quad k \geq 0.$$

Compare the results with the fixed point iteration schemes

$$g_1(x) = \log(x\, e^x),\ g_2(x) = \frac{e^x + x}{e^x + 1},\ g_3(x) = \frac{x^2 - x + 1}{x}$$

by checking the convergence evaluating their first derivatives at the zero $x = 1$.

Take as initial values $x_0 = 2$, $tol = 10^{-10}$ and $nmax = 100$.

Time: **2 hours**.