



Universita' degli Studi di PADOVA

Progetti di Ricerca di Ateneo

Anno: 2012 - prot. CPDA124755

### 1.0 Macroarea di Afferenza del Responsabile Scientifico del Programma di Ricerca

1 - Matematica, scienze fisiche, dell'informazione e della comunicazione, ingegneria e scienze della Terra

### 1.1 Area Scientifica del Responsabile Scientifico del Programma di Ricerca

01 - Scienze Matematiche

### 1.2 Responsabile Scientifico del Programma di Ricerca

|                       |                                    |                                  |
|-----------------------|------------------------------------|----------------------------------|
| DE MARCHI             | Stefano                            | M                                |
| (Cognome)             | (Nome)                             | (sesso)                          |
| PROFESSORE ASSOCIATO  | MAT/08                             | 17/12/1962                       |
| (Qualifica)           | (Settore Scientifico Disciplinare) | (Data di Nascita)                |
| DMRSFN62T17B589H      | Facoltà di SCIENZE STATISTICHE     | DIP. MATEMATICA                  |
| (Codice fiscale)      | (Facoltà)                          | (Dipartimento/Istituto)          |
| +39 0498271425        | +39 0498271499                     | demarchi@math.unipd.it           |
| (Prefisso e Telefono) | (Numero Fax)                       | (Indirizzo di Posta Elettronica) |

### Lingua di compilazione del progetto (inglese o lingua veicolare)

English

### 1.3 Area Scientifica del Programma di Ricerca

|                                    |                     |                  |     |
|------------------------------------|---------------------|------------------|-----|
| <b>Area Scientifica Prevalente</b> | Scienze Matematiche | (% di afferenza) | 100 |
| <b>Area Scientifica</b>            |                     | (% di afferenza) |     |
| <b>Area Scientifica</b>            |                     | (% di afferenza) |     |
| <b>Progetto Interarea</b>          | NO                  |                  |     |

### 1.4 Titolo del Programma di Ricerca

Multivariate approximation with application to image reconstruction

### 1.5 Abstract del Programma di Ricerca

The proposing research group, which includes some components of the Padova-Verona Research Group on Constructive Approximation and Applications (CAA) and other colleagues of the Department of Mathematics, has been working for some years on two wide topics of the theory and algorithms of multivariate approximation: interpolation/approximation by polynomials [L14-1:7, L14-9:11, L14-19:22], and interpolation/approximation by RBF (Radial Basis Functions) or more generally by kernel methods [L5-3].

Particular attention to applications has been always done [L14:5, L14-21,L5-1, L6-3, L6-4]. An interesting multidisciplinary application is image reconstruction and related problems. Kernel methods have recently shown their applicability to image reconstruction, thanks to their flexibility and good converge properties [L5-1, L5-6].

The present project has the following main goals:

- a) to continue the study of polynomial meshes and discrete extremal sets (Fekete, Leja, Lebesgue points) for multivariate polynomial interpolation/approximation (see the research lines L1-L2 in section 2.4);
- b) to develop applications of a), for example in the field of approximation and quadrature on subregions of the sphere, and of discretization of PDEs by spectral and high-order methods (see the research lines L3-L4 in section 2.4);
- c) the application of kernel methods to (medical) image reconstruction will be one of the applications to which we want to deserve attention deepening it both from the analytical and numerical aspects (see research line L5);
- d) it is always important to look for numerical techniques for a "clever" treatment of ill-conditioning of large-scale linear systems. These type of systems often arise in the context of our applications. This is what we want to explore in the research line L6.

These research topics will be studied also within some already well established international collaborations: Claude Brezinski (Lille, France), Armin Iske (Hamburg, Germany), Norm Levenberg (Bloomington, USA) and Gerard Meurant (Paris, France).

In addition, we shall pay particular attention to the production of efficient and reliable numerical software, to allow an effective use of the proposed methods in scientific and technological applications. For an overview of former numerical codes see the web pages <http://www.math.unipd.it/~marcov/CAAssoft.html> and <http://numlab.math.unipd.it/>.

For a complete bibliography of CAA-research group we refer to <http://www.math.unipd.it/~marcov/CAA.html>. For other local components to their personal web pages.

Note: the references here cited are those of section 2.4.

## 1.6 Caratteri di innovatività del progetto e del gruppo

1) In the field of multivariate polynomial approximation a new perspective has been recently opened by the theory of "admissible polynomial meshes" introduced by Calvi and Levenberg in the seminal paper [L14-8]. These meshes are near-optimal for least squares approximation, and contain interpolation sets (discrete extremal sets) which distribute asymptotically as Fekete points of the domain (points that maximize the Vandermonde determinant). Briefly, Admissible Meshes (AM) are sequences  $\{A(n)\}_n$  of discrete "norming" sets for polynomials on a compact  $K$  in  $\mathbb{R}^d$ , i.e., such that  $\|p\|_K \leq C \|p\|_{A(n)}$  for every  $d$ -variate polynomial  $p$  of degree not exceeding  $n$ , with polynomially increasing cardinality in  $n$ , where  $\|f\|_X$  is the sup-norm on a compact  $X$ . In the case when  $C=C(n)$  is not constant but grows at most polynomially in  $n$ , we speak of Weakly Admissible Meshes (WAM). It is worth mentioning, that low cardinality and optimal meshes have been constructed on different domains and dimensions. In one real dimension, we know AM and WAMs on the interval. In the complex plane (weakly) admissible meshes can be readily constructed for any compact which is a (not necessarily disjoint) union of compact sets of the following three types: a segment  $[a; b]$  where  $a$  and  $b$  are two distinct complex numbers; a circle  $C(a; r)$  centered in  $a$  of radius  $r$  (or a closed disk  $D(a; r)$ ); or a general compact set bounded by a smooth Jordan curve. For compacts  $K$  subsets of  $\mathbb{R}^2$  one can construct (W)AMs for all classical domains such as triangles, squares, polygons, disk [L14-3, L14-5, L14-16]. In particular on the triangle, a new set of optimal Lebesgue Gauss-Lobatto points have been recently investigated (cf. [L14-5]). In three dimensional case, apart from tensorial domains and the results here presented, very few is known so far. Therefore one goal will be the study and the construction of (W)AM in higher dimensions and their application to interpolation, cubature and differential equations.

2) The use of (positive definite) kernel methods in medical image reconstruction from scattered Radon data is a recent research line (see [L5-1, L5-6]). During the proposed research we do wish to investigate how to extend this idea to general image reconstruction or other inverse problems.

3) The possibility of using numerical methods for matrix functions for solving ill-conditioned problems has been recently explored in [L6-9], providing a novel approach.

While the relation between the regularized and the exact solution can clearly be written in term of a matrix function, the existing approaches based on the iterative refinement (such as e.g. the Riley's algorithm and the preconditioned Landweber iteration) are based on the computation of its Taylor series with obvious disadvantages. Therefore, the recent developments in the computation of matrix functions have provided new reliable tools and theoretical opportunities for the numerical treatment of ill-posed problems. The use of these tools in this setting will require to define the regularization strategy (regularization parameter and, possibly, regularization matrix) in a different way, as discussed in [L6-6, L6-7].

4) The research group joins together researchers from Italy and abroad, presenting a well established experience from different subjects of numerical analysis (multivariate interpolation/approximation, inverse problems, kernel methods, extrapolation methods, regularization, matrix functions) with the common aim of studying reliable methods that can be used as tools for inverse problems (in principal, image reconstruction) and particular problems from approximation theory. The interdisciplinary nature of the group seems to be a good opportunity to share knowledge and experience, and to produce useful results, as demonstrated in the recent future by the number of publications on these topics.

Note: the cited references can be found in the bibliography of section 2.4 (Program description).

## 1.7 Settori scientifico-disciplinari interessati dal Programma di Ricerca

MAT/08

## 1.8 Parole chiave

1. AREA 01 - Mathematics - Approximations And Expansions - INTERPOLATION
2. AREA 11 - Information Eng. (Mathematics) - Numerical Analysis - Numerical Linear Algebra - ILL-POSEDNESS, REGULARIZATION
3. AREA 11 - Information Eng. (Computer sciences) - Mathematics Of Computing - Mathematical Software - ALGORITHM DESIGN AND ANALYSIS
4. Multivariate polynomial and non polynomial approximation, image reconstruction

## 1.9 Curriculum scientifico del Responsabile Scientifico del programma di ricerca

Born on December 17, 1962 in Candiana (Pd), Italy

1987: degree in Mathematics-University of Padova

1994: Ph.D in Computational Mathematics and Computer Science, VI-cycle Consortium NE, Padua

1995-2005: assistant professor in Numerical Analysis

2005-: associate professor in Numerical Analysis

\* Member of the organizing and scientific committees of the 1st, 2nd and 3rd Dolomites Workshop on Constructive Approximation and Applications (DWCAA06-09-12), Alba di Canazei (Trento, Italy), September 2006, 2009, 2012

\* Member of the organizing and scientific committees of the Dolomites Research Week on Approximation 2007, 2008, 2010, 2011, Alba di Canazei (Trento, Italy)

\* Managing editor of the electronic journal Dolomites Research Notes on Approximation, indexed in Zentralblatt für Mathematik and Open J-Gate

\* Guest Editor of the Proceedings of DWCAA06 and DWCAA09, special issues of Numerical Algorithms

\* Guest editor of the Proceedings "Kernel Functions and Meshless Methods", special issue of DRNA Vol. 4, 2011.

\* Organizer and member of the scientific committee of the "Dolomites Research Week on Approximation", in 2007, 2008, 2010 and 2011.

\* Referee work for: Siam Matrix Analysis and Applications, J. Approx. Theory, Adv. Comput. Math., Numer. Math., J. Comput. Appl. Math, Numer. Algorithms, Med. J. of Math., Proc. A Royal Mathematical Society.

\* Reviewer of the AMS Mathematical Review.

Visiting positions

- Department of Mathematics and Statistics, University of Calgary (Canada): Oct. 1997; Nov. 1999, Nov. 2004.

- Department of Mathematics, University of Dortmund (Germany), Oct. 1998, May-June 1999.

- Department of Mathematics, University of Giessen (Germany), Dec. 1999 with the DAAD scholarship.

- University of Goettingen (Germany), June-July 2001, Aug. 2001, Jan. 2002, Aug. 2002 under the program Vigoni CRUI-DAAD 2001-2002.

- Department of Mathematics, University of Auckland (New Zealand), Feb. 2004.

- Department of Mathematics, University of Zaragoza (Spain), Dec. 2009.

- Department of Mathematics, University of Hamburg (Germany), May 2010 and Jan. 2011.

Coordination of exchange programs

- Vigoni-Program of CRUI in 2001/02
- Bilateral exchange between CNR and DFG German, in 2006.
- Erasmus Teaching Staff Mobility, 2008 and 2011.

Scientific publications

- 50 papers on referred journals, 14 papers on referred proceedings, 1 translation paper, 2 monographs, the PhD thesis and software (see below).

His 5 most cited papers

\*L. Bos, M. Caliarì, S. De Marchi, M. Vianello, Y. Xu: Bivariate Lagrange interpolation at the Padua points: the generating curve approach, *J. Approx. Theory* 143 (1) (2006), 15-25.

classified in the TOP 25 Hottest Articles of JAT, Oct.-Dec. 2006.

\* M. Caliarì, S. De Marchi, M. Vianello: Bivariate polynomial interpolation on the square at new nodal sets,

*Appl. Math. Comput.* 165 (2) (2005), 261-274

\* S. De Marchi, R. Schaback, H. Wendland:

Near-optimal data-independent point locations for radial basis function interpolation,

*Adv. Comput. Math.* 23 (3) (2005), 317-330

\* L. Bos, S. De Marchi, M. Vianello, Y. Xu: Bivariate Lagrange interpolation at the Padua points: the ideal theory approach

*Numer. Mathematik* 108 (1) (2007), 43-57.

\* L. Bos, S. De Marchi, A. Sommariva, M. Vianello: Computing multivariate Fekete and Leja points by numerical linear algebra

*SIAM J. Numer. Anal.* 48 (2010), 1984-1999.

Software published in the Netlib

\* LABSUP: a LABORatory for bivariate C1 Surfaces and Patches,

*Numer. Algorithms* 20(2-3) (1999) (with D. Fasoli and M. Morandi-Cecchi).

\* Padua2D: Lagrange Interpolation at Padua Points on Bivariate Domains, *ACM TOMS* 35-3 (2008) (with M. Caliarì, S. De Marchi)

a variant has been used in Fun2D of the CP2K simulation package for molecular dynamics

\* Padua2DM: fast interpolation and cubature at the Padua points in Matlab/Octave, *Numer. Algorithms* 56 (2011) (with M. Caliarì, S. De Marchi, A. Sommariva)

Complete list of publication at <http://www.math.unipd.it/~demarchi>

## 1.10 Pubblicazioni scientifiche più significative del Responsabile Scientifico del Programma di Ricerca

| n° | Publicazione  |
|----|---|
| 1. | DE MARCHI STEFANO, Martina Marchioro, Alvise Sommariva (2012). Polynomial approximation and cubature at approximate Fekete and Leja points of the cylinder. <i>APPLIED MATHEMATICS AND COMPUTATION</i> , vol. 218; p. 10617-10629, ISSN: 0096-3003, doi: 10.1016/j.amc.2012.04.023 ( <i>Articolo su rivista</i> ) impact factor: 1.536  |
| 2. | Bos Len, DE MARCHI STEFANO, Hormann Kai, Klein Georges (2012). On the Lebesgue constant of barycentric rational interpolation at equidistant nodes. <i>NUMERISCHE MATHEMATIK</i> , vol. 121; p. 461-471, ISSN: 0029-599X, doi: 10.1007/s00211-011-0442-8 ( <i>Articolo su rivista</i> ) impact factor: 1.388  |
| 3. | M. CALIARI, DE MARCHI STEFANO, A. SOMMARIVA, M. VIANELLO (2011). Padua2DM: fast interpolation and cubature at the Padua points in Matlab/Octave. <i>NUMERICAL ALGORITHMS</i> , vol. 56; p. 45-60, ISSN: 1017-1398, doi: 10.1007/s11075-010-9373-1 ( <i>Articolo su rivista</i> ) impact factor: .784 - <i>We have implemented in Matlab/Octave two fast algorithms for bivariate Lagrange interpolation at the so-called Padua points on rectangles, and the corresponding versions for algebraic cubature.</i> |
| 4. | L. Bos, DE MARCHI STEFANO, A. Sommariva, M. Vianello (2010). Computing multivariate Fekete and Leja points by numerical linear algebra. <i>SIAM JOURNAL ON NUMERICAL ANALYSIS</i> , vol. 48; p. 1984-1999, ISSN: 0036-1429, doi: 10.1137/090779024 ( <i>Articolo su rivista</i> ) impact factor: 1.664  |
| 5. | DE MARCHI STEFANO, ROBERT SCHABACK (2010). Stability of Kernel-Based Interpolation. <i>ADVANCES IN COMPUTATIONAL MATHEMATICS</i> , vol. 32; p. 155-161, ISSN: 1019-7168, doi: 10.1007/s10444-008-9093-4 ( <i>Articolo su rivista</i> ) impact factor: 1.438   |

<br clear = all>

## 1.11 Componenti il Gruppo di Ricerca

### 1.11.0 Professori e ricercatori anche a tempo determinato dell'Università di Padova

| n° | Cognome       | Nome    | Dipartimento/Istituto | Area scientifica di ateneo | Qualifica       | Settore | Mesi/Persona(*)<br>Primo anno | Mesi/Persona(*)<br>Secondo anno | Stato della risposta |
|----|---------------|---------|-----------------------|----------------------------|-----------------|---------|-------------------------------|---------------------------------|----------------------|
| 1. | NOVATI        | Paolo   | DIP. MATEMATICA       | 01 - Mathematics           | Ricercatore     | MAT/08  | 6                             | 6                               |                      |
| 2. | REDIVO ZAGLIA | Michela | DIP. MATEMATICA       | 01 - Mathematics           | Prof. Associato | MAT/08  | 6                             | 6                               |                      |
| 3. | SOMMARIVA     | Alvise  | DIP. MATEMATICA       | 01 - Mathematics           | Ricercatore     | MAT/08  | 6                             | 6                               |                      |
| 4. | VIANELLO      | Marco   | DIP. MATEMATICA       | 01 - Mathematics           | Prof. Associato | MAT/08  | 6                             | 6                               |                      |
| 5. | DE MARCHI     | Stefano | DIP. MATEMATICA       | 01 - Mathematics           | Prof. Associato | MAT/08  | 8                             | 8                               |                      |

### 1.11.1 Professori a contratto di cui all'art. 23 della legge 240/2010, altro Personale dell'Università di Padova anche a tempo determinato (personale tecnico-amministrativo, Dirigenti e CEL)

| n° | Nome | Dipartimento/Istituto | Qualifica | Mesi/Persona(*)<br>Primo anno | Mesi/Persona(*)<br>Secondo anno |
|----|------|-----------------------|-----------|-------------------------------|---------------------------------|
|----|------|-----------------------|-----------|-------------------------------|---------------------------------|

### 1.11.2 Titolari di assegni di ricerca dell'Università di Padova

| n° | Cognome | Nome | Dipartimento/Istituto | Area scientifica di ateneo | Mesi/Persona(*)<br>Primo anno | Mesi/Persona(*)<br>Secondo anno |
|----|---------|------|-----------------------|----------------------------|-------------------------------|---------------------------------|
|----|---------|------|-----------------------|----------------------------|-------------------------------|---------------------------------|

### 1.11.3 Studenti di Dottorato di Ricerca dell'Università di Padova

| n° | Cognome    | Nome    | Dipartimento/Istituto | Area scientifica di ateneo | Qualifica  | Mesi/Persona(*)<br>Primo anno | Mesi/Persona(*)<br>Secondo anno |
|----|------------|---------|-----------------------|----------------------------|------------|-------------------------------|---------------------------------|
| 1. | GAZZOLA    | Silvia  | DIP. MATEMATICA       | 01 - Scienze Matematiche   | Dottorando | 4                             | 4                               |
| 2. | KARAPIPERI | Anna    | DIP. MATEMATICA       | 01 - Scienze Matematiche   | Dottorando | 4                             | 4                               |
| 3. | POZZA      | Stefano | DIP. MATEMATICA       | 01 - Scienze Matematiche   | Dottorando | 4                             | 4                               |

### 1.11.4 Professori, ricercatori (anche a tempo determinato), dottorandi e assegnisti di altre Università

| n° | Cognome | Nome          | Università                       | Area scientifica di ateneo | Dipartimento/Istituto | Qualifica       | Settore | Mesi/Persona(*)<br>Primo anno | Mesi/Persona(*)<br>Secondo anno |
|----|---------|---------------|----------------------------------|----------------------------|-----------------------|-----------------|---------|-------------------------------|---------------------------------|
| 1. | BOS     | Leonard Peter | Università degli Studi di VERONA | 01 - Scienze Matematiche   | DIP. INFORMATICA      | Prof. Ordinario | MAT/08  | 6                             | 6                               |

### 1.11.5 Dipendenti di altre amministrazioni pubbliche, di enti pubblici o privati, di imprese, di istituzioni straniere, soggetti esterni in possesso di specifiche competenze nel campo della ricerca

| n° | Cognome   | Nome     | Ente  | Qualifica       | Mesi/Persona(*)<br>Primo anno | Mesi/Persona(*)<br>Secondo anno |
|----|-----------|----------|---|-----------------|-------------------------------|---------------------------------|
| 1. | BREZINSKI | CLAUDE   | University of Lille (France)                          | Emeritus        | 4                             | 4                               |
| 2. | ISKE      | ARMIN    | University of Hamburg (Germany)                       | Full Professor  | 4                             | 4                               |
| 3. | LEVENBERG | NORMAN   | Indiana University Bloomington (United States)        | Lecturer        | 4                             | 4                               |
| 4. | MEURANT   | GERARD   | Commissariat a l'Energie Atomique DIF (France)        | Retired         | 4                             | 4                               |
| 5. | SIRONI    | AMOS     | Ecole Polytechnique Federal de Lausanne (Switzerland) | Ph.D. student   | 3                             | 3                               |
| 6. | SANTIN    | GABRIELE | University of Padova                                  | Master's degree | 3                             | 3                               |

### 2.1.0 Pubblicazioni scientifiche più significative dei componenti il gruppo di ricerca (docenti dell'ateneo di Padova)

| n° | Pubblicazioni  |
|----|--|
| 1. | G. Da Fies, VIANELLO M. (2012). Trigonometric Gaussian quadrature on subintervals of the period. ELECTRONIC TRANSACTIONS ON NUMERICAL ANALYSIS. vol. 39, pp. 102-112 ISSN: 1068-9613.  |
| 2. | M. GENTILE, SOMMARIVA A., M. VIANELLO. (2011). Polynomial interpolation and cubature over polygons. JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS. vol. 235, pp. 5232-5239 ISSN: 0377-0427. doi:10.1016/j.cam.2011.05.013.  |
| 3. | F. Rapetti, SOMMARIVA A., M. Vianello. (2011). On the generation of symmetric Lebesgue-like points in the triangle. JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS. pp. - ISSN: 0377-0427. doi:10.1016/j.cam.2011.11.023.  |
| 4. | BREZINSKI C, REDIVO ZAGLIA M. (2011). Pade'-type rational and barycentric interpolation. arXiv:1107.4854v1. (pp. 1-22). <a href="http://arxiv.org/abs/1107.4854">http://arxiv.org/abs/1107.4854</a> .  |
| 5. | BOS L, CALVI, J.P, LEVENBERG N, SOMMARIVA A, VIANELLO M. (2011). Geometric Weakly Admissible Meshes, Discrete Least Squares Approximations and Approximate Fekete Points. MATHEMATICS OF COMPUTATION. vol. 80, pp. 1601-1621 ISSN: 0025-5718. doi:10.1090/S0025-5718-2011-02442-7. |
| 6. | MORET I, NOVATI P. (2011). On the convergence of Krylov subspace methods for matrix Mittag-Leffler functions. SIAM JOURNAL ON NUMERICAL ANALYSIS. vol. 49, pp. 2144-2164 ISSN: 0036-1429, 000296605600019 doi:10.1137/080738374.   |
| 7. | BREZINSKI C, HE Y, HU X-B, REDIVO ZAGLIA M., SUN J-Q. (2012). Multistep epsilon-algorithm, Shanks' transformation, and the Lotka-Volterra system by Hirota's method. MATHEMATICS OF COMPUTATION. vol. 81, pp. 1527-1549 ISSN: 0025-5718, 000305099000012                           |

|     |  |
|-----|--|
|     | doi:10.1090/S0025-5718-2011-02554-8.   |
| 8.  | NOVATI P. (2011). Using the Restricted-Denominator rational Arnoldi method for Exponential integrators. SIAM JOURNAL ON MATRIX ANALYSIS AND APPLICATIONS. vol. 32, pp. 1537-1558 ISSN: 0895-4798, 000298373400023 doi:10.1137/100814202.                   |
| 9.  | BREZINSKI C, NOVATI P., REDIVO ZAGLIA M. (2012). A rational Arnoldi approach for ill-conditioned linear systems. JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS. vol. 236, pp. 2063-2077 ISSN: 0377-0427, 000300031400003 doi:10.1016/j.cam.2011.09.032. |
| 10. | REDIVO ZAGLIA M., RODRIGUEZ G. (2012). smt: a Matlab toolbox for structured matrices. NUMERICAL ALGORITHMS. vol. 59, pp. 639-659 ISSN: 1017-1398, 000302380300009 doi:10.1007/s11075-011-9527-9.   |
| 11. | L. Bos, S. De Marchi, A. Sommariva, VIANELLO M. (2010). Computing multivariate Fekete and Leja points by numerical linear algebra. SIAM JOURNAL ON NUMERICAL ANALYSIS. vol. 48, pp. 1984-1999 ISSN: 0036-1429. doi:10.1137/090779024.                      |
| 12. | Matteo Briani, SOMMARIVA A., Marco Vianello. (2012). Computing Fekete and Lebesgue points: Simplex, square, disk. JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS. vol. 236, pp. 2477-2486 ISSN: 0377-0427. doi:10.1016/j.cam.2011.12.006.                |

## 2.1.1 Pubblicazioni scientifiche più significative dei componenti il gruppo di ricerca (altri partecipanti al progetto)

[B1] L. Bos and M. Vianello  
Subperiodic trigonometric interpolation and quadrature  
Appl. Math. Comput. 218 (2012), 10630--10638

[B2] L. Bos and M. Vianello  
Low cardinality admissible meshes on quadrangles, triangles and disks  
Math. Inequal. Appl. 15 (2012), 229--235

[B3] L. Bos, S. De Marchi, A. Sommariva and M. Vianello  
On Multivariate Newton Interpolation at Discrete Leja Points  
Dolomites Res. Notes Approx. DRNA 4 (2011), 15--20

[BR1] C. Brezinski  
From numerical quadrature to Pad' approximation.  
Appl. Numer. Math., 60 (2010) 1209--1220.

[BR2] C. Brezinski, G. Rodriguez, S. Seatzu  
Error estimates for linear systems with applications to regularization (avec .  
Numer. Algorithms, 49 (2008), 85--104.

[BR3] C. Brezinski  
Projection Methods for Systems of Equations.  
North-Holland, Amsterdam, 1997, VII + 400 pages.

[I1] Tim Nielsen, Sebastian Hitziger, Michael Grass, and Armin Iske  
Filter calculation for x-ray tomosynthesis reconstruction.  
Physics in Medicine and Biology 57, 2012, 3915-3930.

[I2] Laurent Demaret and Armin Iske  
Optimal N-term Approximation by Linear Splines over Anisotropic Delaunay Triangulations.  
Preprint, 28th March 2012.

[I3] G. Plonka, S. Tenorth, and A. Iske  
Optimally Sparse Image Representation by the Easy Path Wavelet Transform. International Journal of Wavelets, Multiresolution and Information Processing (IJWMIP) 10(1), 2012, 1250007 (20 pages).

[L1] J.-P. Calvi, N. Levenberg, Uniform approximation by discrete least squares polynomials, J. Approx. Theory 152 (2008).

[L2] L. Bos, J.-P. Calvi, N. Levenberg, A. Sommariva, M. Vianello, Geometric Weakly Admissible Meshes, Discrete Least Squares Approximation and Approximate Fekete Points, Math. Comp. 80 (2011).

[L3] T. Bloom, L. Bos and N. Levenberg  
The Asymptotics of Optimal Designs for Polynomial Regression  
submitted (2012)

[M1] G. Meurant, Computer solution of large linear systems. North-Holland (1999)

[M2] G.H. Golub & G. Meurant, Matrices, moments and quadrature with applications. Princeton University Press (2010).

[M3] G. Meurant and A. Sommariva, Fast variants of the Golub and Welsch algorithm for symmetric weight functions, submitted (2012).

[Sir1] S. De Marchi, A. Iske and A. Sironi  
Algebraic Medical Image Reconstruction from Scattered Radon Data by Positive Definite Kernels, Submitted 2012.

[San1] S. De Marchi and G. Santin  
A new stable basis for RBF interpolation  
Poster accepted and to be present at the DWCAA12, Alba di Canazei 9-14 Sept. 2012.

[San2] G. Santin, A. Sommariva and M. Vianello  
An algebraic cubature formula on curvilinear polygons  
Appl. Math. Comput. 217 (2011), 10003--10015

## 2.2 Curriculum scientifico dei Componenti il Gruppo di Ricerca

Paolo NOVATI

Paolo Novati got his Laurea Degree in Applied Mathematics at the University of Trieste - Italy in 1996 and his Ph.D. in Computational Mathematics in 2001, at the School of Mathematics of the University of Padova - Italy. Soon after, he got a post-doc position at the University of Trieste. On December 2001 he became Assistant Professor of Numerical Analysis at the University of L'Aquila - Italy. On December 2009 he moved to the University of Padova, Department of Pure and Applied Mathematics. From 2003 to 2009 he was also Teaching Assistant of Analysis, Linear Algebra, Geometry, at the Faculty of Engineering, Nettuno University, Rome - Italy. Paolo Novati is author or co-author of more than 25 papers (9 as single author) in several high qualified international journals. His research is mainly devoted to the following topics: matrix functions, ordinary and fractional differential equations, regularization of ill-posed linear systems. He participated to various national research projects such as PRIN and GNCS. He is member of the scientific committee of the Ph.D. in Computational Mathematics of the University of Padova. He is currently in charge of a two-years scientific project of the University of Padova on the numerical treatment of ill-posed linear problems. On this last topic, in 2011 he presented a research program entitled "Regularization techniques based on Krylov methods for ill-posed linear systems" that won a research fellowship for Ph.D. students in Computational Mathematics (3-years grant funded by the Fondazione Cassa di Risparmio di Padova e Rovigo, started in 2011).

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Michela REDIVO ZAGLIA

Associate Professor in Numerical Analysis since 1998. From 1998 to 2002, at the Dep. of Mathematics of the University of Calabria then at the Dep. of Pure and Applied Mathematics of the University of Padua. Head of the Computer Center of the Department of Electronics and Computer Sciences the University of Padua from 1984 to 1998.

Visiting Professor in several European Universities and also in Africa (Johannesburg and Marrakech) giving there more than 30 seminars. Her scientific works has been presented at about 40 international conferences. Member of the Scientific and Organizing Committee of 15 Conferences and Editor of 9 conference proceedings. She attended, also as invited speaker, to several international conferences related to her areas of research. Member of the editorial board of 4 international journals: Int. J. of Appl. Math. and Eng. Sciences, J. of Appl. Math., Dolomites Res. Notes Approx. and Numer. Algorithms. For the last one she is also the software editor since 1991. Member of Ph.D. examination committees in Italy, France, Portugal and Morocco and member of the board of the Doctoral School in Computational Mathematics at University of Padua since 2005.

From 2008 to 2011, she was vice-director of the Department of Pure and Applied Mathematics of Padova University.

Main interests in numerical analysis: extrapolation methods and convergence acceleration, classical and formal orthogonal polynomials; solution of systems of equations; interpolation and approximation methods and algorithms; regularization (inverse problems); PageRank algorithm (Google's web search); numerical linear algebra; numerical software.

Author or coauthor of about 70 papers in international journals and conference proceedings and seven books (one of which in English, published in 1991 by North-Holland, and two in French). Another book is in preparation.

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Alvise SOMMARIVA

- Born in Venice, October 11, 1968.

- Degree in Mathematics, University of Padua.

- Ph. D. in Computational Mathematics (1996-1999).

- Research activity, project "MAST III PACE", University of Rome III (1999).

- Research Associate, project "Numerical Analysis of integral and differential models in applied sciences", University of Padua (1999-2002).

- Post-doc fellowship, University of Padua (2002-2004).

- Research Associate, University of New South Wales, Australia ( September 1, 2004 - November 31, 2005).

- Assistant Professor in Numerical Analysis: University of Padua (March 2006-).

- Referee for "Numerische Mathematik", "IMA Journal Numerical Analysis", "Journal of Integral Equations and Applications", "Electronic Transactions in Numerical Analysis", "Computing", "Applied Numerical Mathematics", "Numerical Algorithms", "Journal Of Computational and Applied Mathematics".

Author of more than 35 papers in approximation theory, cubature, partial differential and integral equations.

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Marco VIANELLO

\* Born in Venice on october 26, 1961.

\* Ph.D. in Computational Mathematics, Univ. of Padova 1992.

\* INdAM research fellowship from nov. 1991 to oct. 1993.

\* Assistant prof. (1993-2000) and associate prof. (2000-) of Numerical Analysis at the Fac. of Science of the Univ. of Padova. RESEARCH RESULTS

\* Author of over 90 papers in the fields of Numerical Analysis and Mathematical Analysis: 1. Multivariate approximation: polynomials, radial basis functions, numerical cubature (30 papers) 2. Approximation of matrix exponentials and exponential integrators for ODEs/PDEs (8 papers) 3. Constructive-numerical methods for integral equations (11 papers) 4. Qualitative and quantitative asymptotics: 2nd order differential and difference eqs./systems, special functions (25 papers) 5. Numerical functional analysis (8 papers) RESEARCH COLLABORATION AND COORDINATION EXPERIENCE

\* Supervisor of 2 Ph.D. theses and of 3 post-doc fellowships in Computational Mathematics.

\* International collaborations with L. Bos (Calgary), J.P. Calvi (Toulouse), W. Gautschi (Purdue), N. Levenberg (Bloomington), F. Rapetti (Nice), S. Waldron (Auckland), Y. Xu (Eugene).

\* Scientific coordinator of 3 research projects (overall grants: about 60000 euros).

\* Member of the CAA: Padova-Verona Research Group on Constructive Approximation and Applications. \* Member of the Council of the Ph.D. program in Computational Mathematics of the Univ. of Padova (2005-2010). CONFERENCE ORGANIZATION AND EDITORIAL EXPERIENCE

\* Member of the organizing and scientific committees of the 1st, 2nd and 3rd Dolomites Workshop on Constructive Approximation and Applications

(DWCAA06-09-12), Dolomites Research Week on Approximation 2007-08-10-11 (DRWA07-08-10-11).

\* Guest editor of the Proc. of DWCAA06 and DWCAA09, special issue of Numer. Algorithms.

\* Managing editor of Dolomites Research Notes on Approximation (DRNA).

## 2.3 Stato dell'Arte: base di partenza scientifica nazionale ed internazionale

The research group brings together the CAA-group (Padova-Verona Research Group on Constructive Approximation and Applications) and other colleagues in numerical analysis with whom we have started a strong scientific collaboration in the last years.

The CAA-group has been working for years on two wide topics of the theory and algorithms of multivariate approximation: interpolation/approximation by polynomials and by Radial Basis Functions (RBF) or generally kernel functions.

For a complete bibliography on the group activity (publications, software, workshops and collaborations) see the web site <http://www.math.unipd.it/~marcov/CAA.html> and <http://numlab.math.unipd.it>.

The research of the whole group takes advantage since some years of an international collaboration network. Many of these collaborations were born or have been reinforced within the conference series Dolomites Workshops on Constructive Approximation and Applications, which are organized yearly since 7 years in September at the summer site of Alba di Canazei (Trento) of the University of Verona (see also the web page <http://numlab.math.unipd.it/>). From these activities in 2008 stemmed also the open-access electronic journal Dolomites Research Notes on Approximation (DRNA), <http://drna.di.univr.it>, now indexed on Zentralblatt MATH.

The proposed project aims to deepen some open problems already studied and partially solved in the last decade by the participants.

(A) Locating good points for multivariate polynomial approximation, in particular for interpolation, is still a difficult open problem (cf. i.e. C. de Boor, Issues in multivariate polynomial interpolation, plenary talk at "Perspectives in Numerical Analysis", Helsinki, May 2008). In fact, it is well known since the discovery of

Runge's phenomenon in one dimension, that the geometry of a discrete model of a compact has a strong influence on the quality of approximation and interpolation based on it. Our main results in this field concern: optimal interpolation on the square (by the so called "Padua points" [L14-1, L14-7]), hyperinterpolation (i.e., discretized expansions in series of multivariate orthogonal polynomials [L14-10]), construction of "admissible" polynomial meshes on various compacts by geometric transformations (they are quasi-optimal meshes for least squares approximation which contain near-optimal nodal sets for interpolation [L14-3, L14-15]), efficient computation of discrete extremal sets (approximate Fekete, Leja and Lebesgue points [L14-2, L14-3, L14-5, L14-16, L14-21]), construction of algebraic cubature formulas on compacts with various geometries [L14-11, L14-19, L14-20].

(B) In the field of approximation by RBF, we have worked mainly on stability of RBF interpolation [L5-2, L5-3], interpolation by RBF plus polynomials [L14-17] "meshfree" cubature with scattered data [L14-18].

Recently, we have started to explore new interesting applications requiring strong mathematical background: the reconstruction of medical images by approximation with positive definite kernels (also from scattered data [L5-1]).

The use of stable algorithms for the construction of kernel based approximation are a key tool in this setting. That is why, for getting good results in image applications, we need to work along these two directions:

- i) the study of new kernel based models;
- ii) the development of stable numerical algorithms especially for large scale ill-conditioned linear systems (as outlined in line L6 of section 2.4.)

As a final aim of the present project is just that of stabilize, reinforce and extend the international collaboration network, by constructing a "reference point" for the activities in the field of constructive approximation.

Note: the references here cited are those of section 2.4 (Program description).

## 2.4 Descrizione del Programma di Ricerca

The proposed program has been organized in 6 research lines corresponding to our recent research contributions, the future works and the open problems related to multivariate approximation (by polynomial and non-polynomial bases) and some of their applications. Both theoretical and numerical aspects will be investigated.

### L1) OPTIMAL POLYNOMIAL MESHES

A new perspective in the field of multivariate polynomial approximation has been recently opened by the theory of "admissible" polynomial meshes [L14-12], that are near-optimal points for least squares approximation, and contain interpolation sets (discrete extremal sets) which distribute asymptotically as Fekete points of the domain (points that maximize the Vandermonde determinant). Starting from our recent work on polynomial meshes and discrete extremal sets (see e.g. [L14-2, L14-3]), we have in mind several research lines and applications, that we briefly describe.

Briefly, Admissible Meshes (AM) are sequences  $\{A(n)\}$  of discrete "norming" sets for polynomials on a compact  $K$  in  $R^d$ , i.e., such that  $\|p\|_K \leq C \|p\|_{A(n)}$  for every  $d$ -variate polynomial  $p$  of degree not exceeding  $n$ , with polynomially increasing cardinality in  $n$ , where  $\|f\|_X$  is the sup-norm on a compact  $X$ .

In the case when  $C=C(n)$  is not constant but grows at most polynomially in  $n$ , we speak of "weakly admissible meshes".

It has been recently proved that there exist "optimal" admissible meshes, i.e. admissible meshes with cardinality  $O(n^d)$ , in multidimensional compacts with various geometries [L14-13]. We plan to deepen the study of optimal admissible meshes, for example for convex compacts, and of three-dimensional weakly admissible meshes.

### L2) COMPUTING EXTREMAL NODAL SETS

Computing Fekete points (points that maximize the Vandermonde determinant) and Lebesgue points (points that minimize the Lebesgue constant) of a multidimensional compact is a challenging numerical problem. Approximate Fekete points [L14-2, L14-3, L14-22], obtained by standard numerical linear algebra methods, are reasonable candidates as initial guess in a nonlinear optimization method [L14-5]. We plan to pursue this computational approach, which could be useful also for conjecturing the possible distribution of such points, which is theoretically known only for one-dimensional Fekete points. In particular it seems interesting, in view of applications to the 3D spectral element method, to compute Fekete and Lebesgue points of the tetrahedron.

### L3) APPLICATION OF EXTREMAL NODAL SETS TO THE NUMERICAL SOLUTION OF PDES

In [L14-21], a first step in the application of polynomial meshes and discrete extremal sets to the numerical solution of PDEs has been made: the program is to extend such an approach to the trivariate TSEM method (spectral element method on tetrahedral meshes). Moreover, approximate Fekete points [L14-2, L14-3] could be directly used for global polynomial approximation of PDEs (like the "magic points" studied in [L14-14]); a first application has appeared in [L14-22].

Even though they are still at a preliminary stage, these topics deserve some attention in view of the relevance of PDEs in applications.

A new insight has been recently given with new fast algorithms for computing Fejér type and Gaussian rules with more than thousands nodes and applied these techniques for determining new product formulas on spherical domains. This will allow to determine new stable rules for univariate and multivariate domains, to solve PDE problems via RBF-Galerkin methods (cf. [L14-18, L14-18.1]).

### L4) SUBPERIODIC TRIGONOMETRIC INTERPOLATION AND QUADRATURE AND APPROXIMATION ON SUBREGIONS OF THE SPHERE

In a quite recent paper we have constructed "subperiodic" (on intervals smaller than the period) interpolation and quadrature formulas for trigonometric polynomials [L14-4]. In this framework it seems natural to study also gaussian-like subperiodic trigonometric quadrature formulas. The idea is to use this approach to construct polynomial meshes, discrete extremal sets and algebraic cubature formulas on various arc-based sections of disk, sphere/ball (of interest in the field of geomathematics [L14-12]), cylinder (already developed in [L14-9]), cone, torus (e.g., sectors, circular segments and lenses, caps, spherical and toroidal rectangles, ...). A possible connection with meshfree projection methods (continuous least squares, meshfree Galerkin-like method) is given by the application of the new quadrature formulas for circular and spherical lenses to the efficient construction of discretization matrices, which needs integration of product of functions with compact radial support.

These researches will be in collaboration with Leonard Peter Bos (Verona), Norman Levenberg (Bloomington) and Gerard Meurant (Paris).

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Polynomial approximation and cubature at approximate Fekete and Leja points of the cylinder, *Appl. Math. Comput.* 218(21) (2012).

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The activity in the field of meshfree approximation by radial bases, solution of ill-conditioned systems (often arising in this setting) and their main application to image reconstruction are described in the following research lines.

#### L5) IMAGE RECONSTRUCTION BY KERNEL BASED METHODS

The problem of image reconstruction, that consists in approximating a function starting from its Radon transform, arises for example in the context of medical imaging when one wants to reconstruct the internal structure starting from a X-ray tomography (SPECT or CAT). Classical reconstruction methods are based on the so-called "filtered back projection formula" or its variants. We propose a novel kernel-based algebraic reconstruction method for medical image reconstruction from scattered Radon data [L5-1, L5-6]. The reconstruction relies on generalized Hermite-Birkhoff interpolation by positive definite kernel functions (see [L5-5] for an overview of positive definite kernel functions) in combination with a suitable regularization of the Radon transform. This leads to a very flexible reconstruction method for medical images, whose good performance is supported by numerical examples and comparisons with classical Fourier-based methods relying on the Iterated Back Projection formula like ART (Algebraic Reconstruction Techniques) (see [L5-4] for a Matlab implementation of such techniques). We want also to study more deeply the behavior of the method, especially for what concerns the error analysis which is a challenging result.

In the context of radial kernels, an important problem is the choice of the basis. As well known, the stability of basis guarantees better approximation properties. In his recent Master's thesis "A new stable basis for RBF interpolation", G. Santin gave convergence estimates and stability bounds for interpolation and least-squares approximation based on a new "natural" basis (cf. [L5-2, L5-3]). We propose to apply this new stable basis to the reconstruction of images by kernels.

This research will be in collaboration with Armin Iske (Hamburg) and Amos Sironi (Lousanne).

#### References

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#### L6) LARGE-SCALE ILL-CONDITIONED LINEAR SYSTEMS

In the framework of the numerical solution of large-scale ill-conditioned linear systems [L6-8] modeling image blurring, we are interested in studying innovative techniques based on Krylov type solvers [L6-5]. Recently, in [L6-2, L6-9] some methods based on the iterated Tikhonov regularization coupled with the computation of suitable matrix function by means of the Arnoldi and Lanczos process have been considered. In [L6-7, L6-8] the single- and multi-parameter Tikhonov regularization projected onto the Krylov subspaces generated by the Arnoldi algorithm have been analyzed and implemented. All the methods presented in this work have been shown to be particularly efficient to face large-scale problems (as for instance image reconstruction) since they allow to work in reduced dimensions. Our proposal is to continue the analysis of these iterative solvers in the following main directions:

- 1) analysis of the different implementations of the Arnoldi based methods and of other well known Krylov solver in the framework of the regularization. In our opinion, in this context there is still some ambiguities for what concerns the use of some well established techniques such as reorthogonalization, hybrid methods,...
- 2) study of new regularization operators able to preserve edges in image reconstruction. Some preliminary experiments, which employ regularization operators defined as the matrices which extract the details associated to some common wavelet decomposition, have been already performed with promising results.
- 3) use of extrapolation methods [L6-1], that gave good results in the standard Tikhonov regularization [L6-3, L6-4], could be modified and/or adapted to the previous approaches.

These aspects will be studied in collaboration with Claude Brezinski (Lille).

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## 2.5 Obiettivo del Programma di Ricerca ed indicazione dei risultati previsti alla fine del primo anno e a conclusione della ricerca

The proposed program has the following main goals

1. To continue the study of polynomial meshes and discrete extremal sets (Fekete, Leja, Lebesgue points) for multivariate polynomial interpolation/approximation, and to develop their applications, for example in the field of approximation and quadrature on subregions of the sphere, and of discretization of PDEs by spectral and high-order methods (as described in the research lines L1-L4).
2. The application of kernel methods to image reconstruction has shown its flexibility. It is based on positive definite kernel functions and it is very flexible thanks to the possibility to choose different kernels and parameters. We want to study more deeply the behavior of the method, especially for what concerns the error analysis, and compare it with classical available algorithms like ART (Algebraic Reconstruction Techniques) as described in the research line L5. In particular the required biennial research grant on "New mathematical tools for medical image reconstruction" (see section 3.1) is aimed to support a post-doc (or a post-master) student who





|  |                  |               |
|--|------------------|---------------|
| <b>Attrezzature scientifiche di importo superiore a 5.000 Euro</b> | (vedi punto 3.3) |               |
| <b>TOTALE</b>  |                  | <b>97.000</b> |

Il presente progetto NON prevede sperimentazione animale

**SI DICHIARA INOLTRE QUANTO SEGUE:**

1) È stata presentata richiesta di finanziamento, per lo stesso o analogo progetto, anche ad altro Ente, da parte del Responsabile o dei componenti il gruppo di ricerca: NO  
 Se sì indicare:

- a quale Ente:

NO

2) La realizzazione del presente progetto sarà sovrapposta alla realizzazione di altri rilevanti progetti di ricerca:  
 Se sì, indicare quali:

Il Responsabile della Ricerca: .....

Il Direttore della Struttura: .....

Per la copia da depositare presso l'Ateneo e per l'assenso alla elaborazione e diffusione delle informazioni riguardanti i programmi di ricerca presentati; decreto legislativo 196/03 sulla "Tutela dei dati personali".

Il Responsabile della Ricerca: .....

Padova li, 30/07/2012 15:05