"Matching food and wine" and (some) mathematics

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Outline

1. Motivation
2. What does matching mean?
3. Some math
4. Time to try!

Motivation

- Sommeliers or wine tasters, in matching foods with wines, use a (graphical) diagram aimed to match as properly as possible a given dish or a simple food with an appropriate wine.
- At the end of this evening we hope that everyone is more confident in choosing the proper wine for almost every dish!

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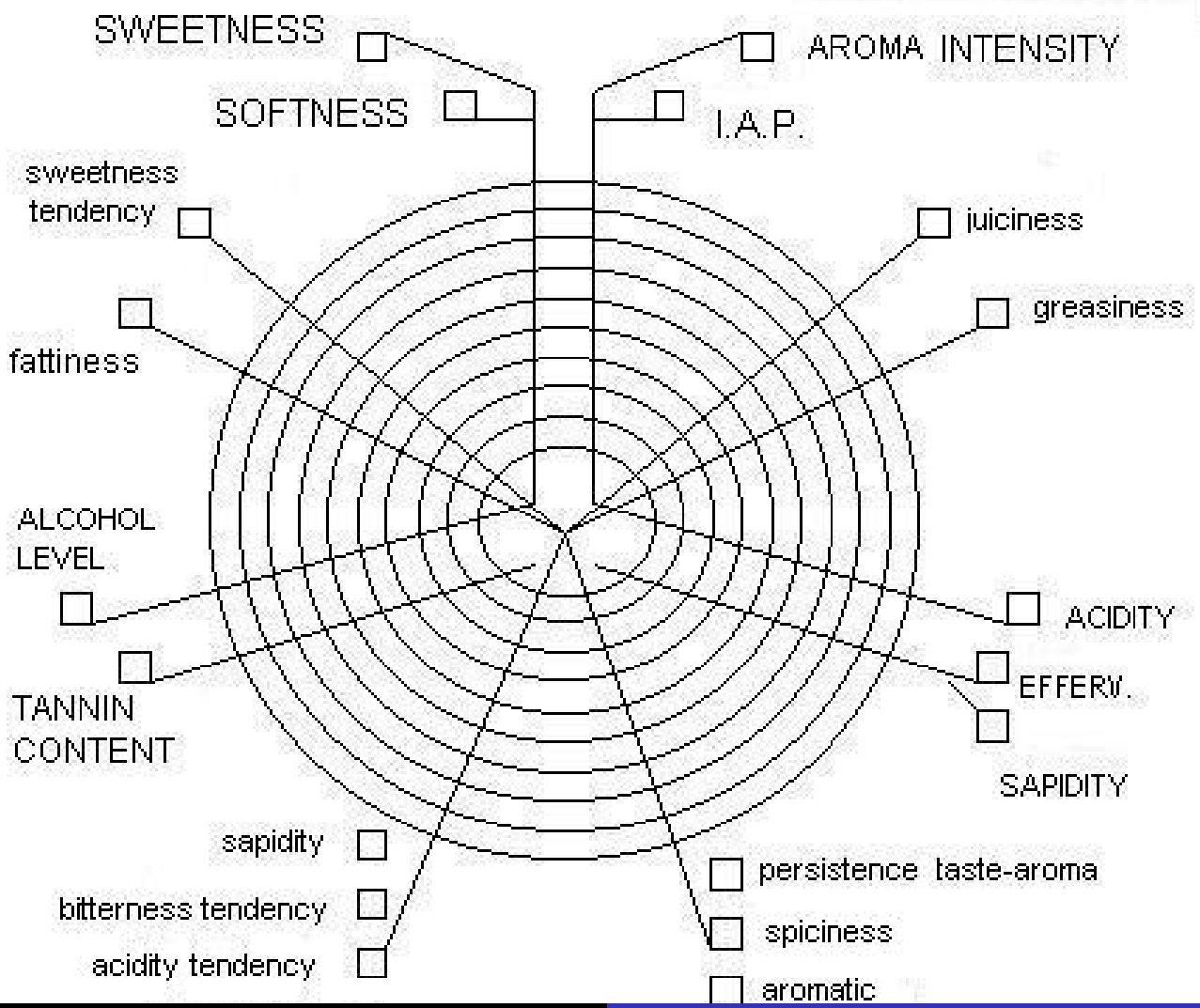
The diagram


SWEETNESS
AROMA INTENSITY
SOFTNESS
I.A.P.
sweetness tendency
juiciness
greasiness
fattiness
ALCOHOL LEVEL
ACIDITY
EFFERV.
TANNIN CONTENT
SAPIDITY
sapidity
persistence taste-aroma
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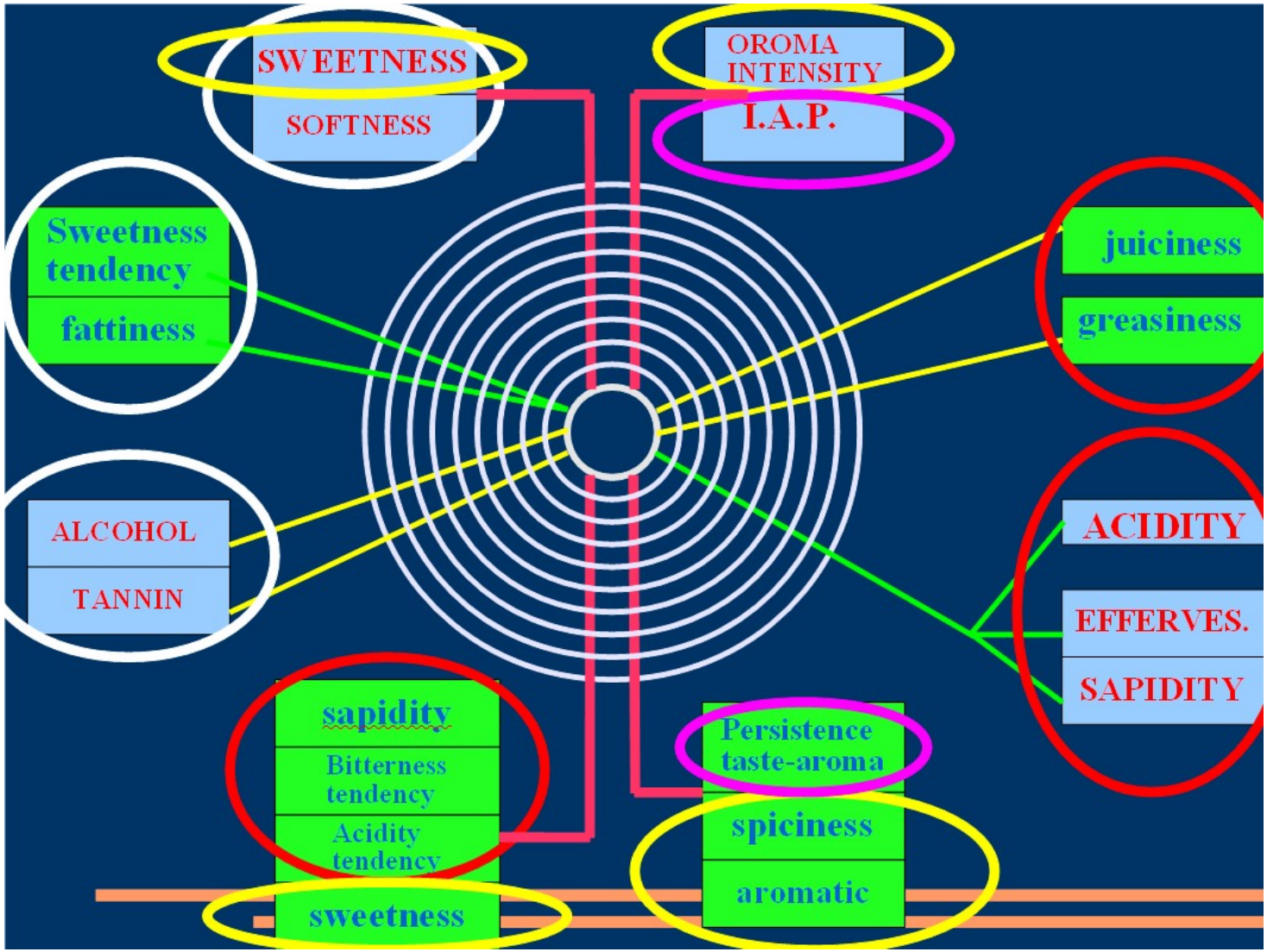
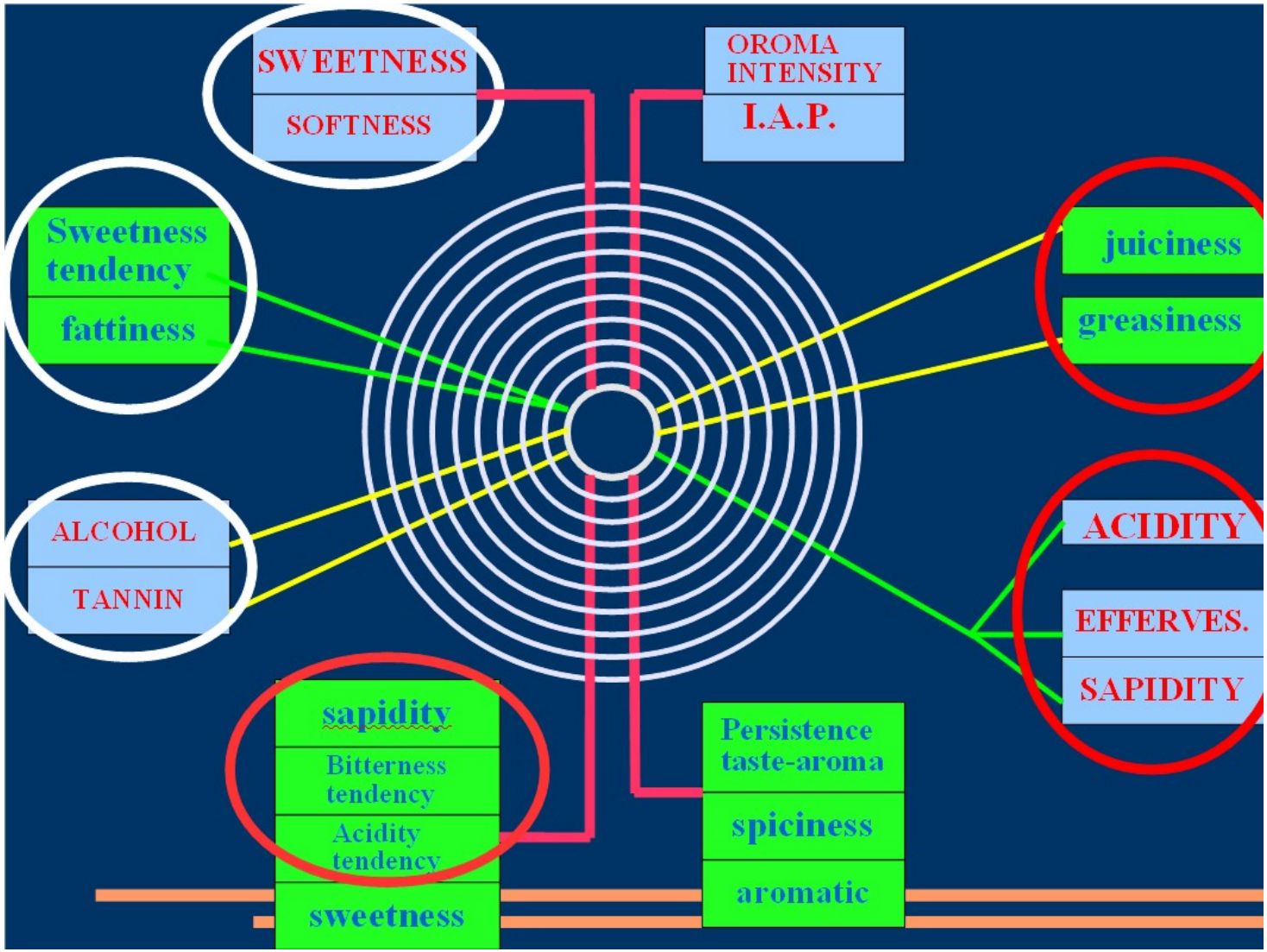
aromatic

More about the diagram



Example 1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Filling the scheme

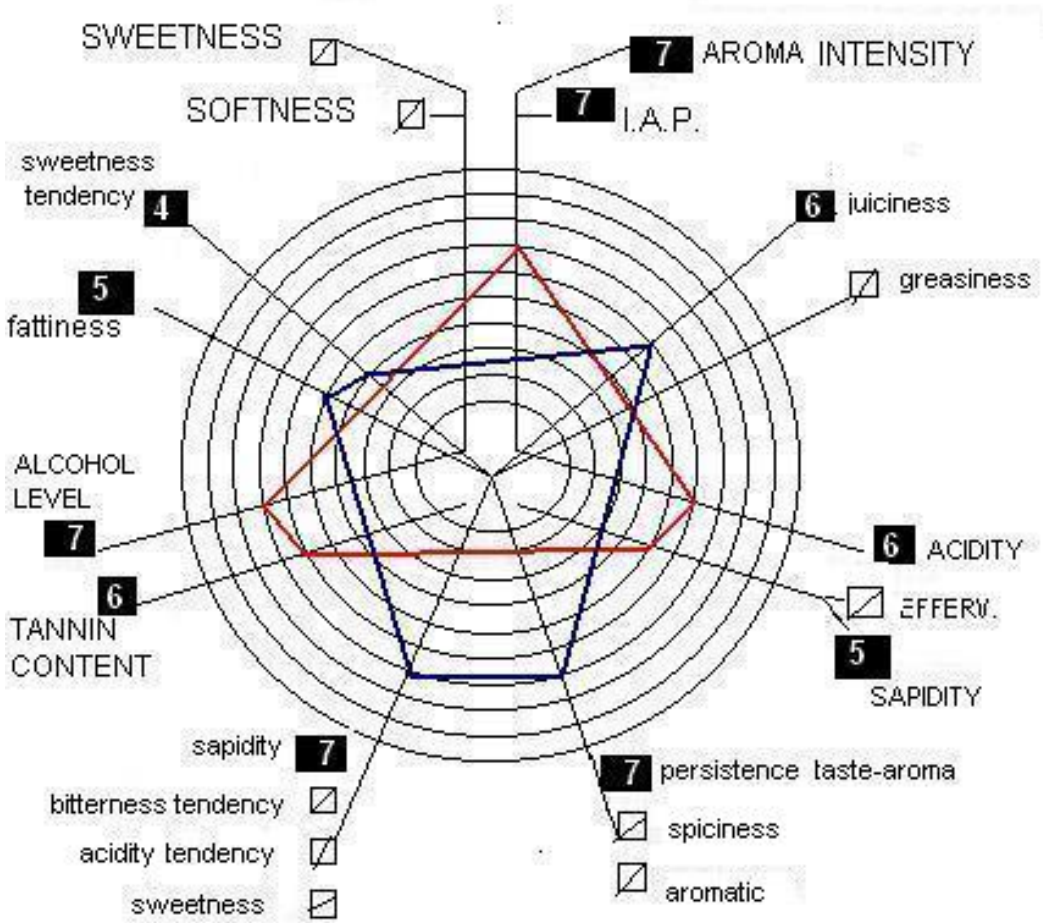
These are the rules (not all!) one has to follow.

- Words in normal size, correspond to the food characteristics that we want to evaluate using a scale from 0 to 10.
- Words in capitals, correspond to the wine characteristics that we want to evaluate, again using a scale from 0 to 10.

1. We use the value 0, when a characteristic is absent.
2. We use values $1-3$, when a characteristic is just perceptible.
3. We use values $4-6$, when a characteristic is better perceptible than before, but not clearly.
4. We use values $7-8$, when a characteristic is perceptible in a good way.
5. We use values $9-10$, when a characteristic is perfectly perceptible.

Example 2

Here we show the diagram for the match of a slice of S. Daniele ham, "prosciutto crudo" (in blue) and a red wine from Sicily DOC Nero d'Avola 2002, 14% (in red).



What does matching mean?

The procedure allows to answer the following 2 questions:

- For given food and wine, is the wine matching or not matching the food?
- Given a food, which characteristics should a wine have for the optimal match (or the best possible)?

Mathematically speaking, this is a kind of proof of existence of the best match (like best interpolation ...)

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What does it mean "matching"?

- From Example 2, the "best matching problem" is a (simple) geometrical problem: a comparison of the areas of 2 polygons!.
- The polygons should be "as similar as possible". Modulo a roto-translation they should (*possibly completely*) overlap!
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Some math

- Consider the center of the circles as the origin!
- Every polygon has vertices at points of the form

$$(x_s, y_s) = (k \cos\theta_s, k \sin\theta_s),$$

where $k \in \{0, 1, \ldots, 10\}$ and θ_s are the angles of the lines.

- For instance, for the wine characteristics we may choose: $\theta_1 = \pi/3$, $\theta_2 = 2\pi/3$, $\theta_3 = 7\pi/6$, $\theta_4 = 5\pi/4$, $\theta_5 = 7\pi/4$, $\theta_6 = 11\pi/6$.
- Similarly for the food, with $s = 1, \ldots, 11$.

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Some maths

- Letting $\mathcal{W} = \{(x_s, y_s),\ s = 1, \ldots, S_W\}$ and $\mathcal{F} = \{(u_s, v_s),\ s = 1, \ldots, S_F\}$ be the two polygons.
- By the discrete version of Green's formula for the area enclosed in a closed curve, we can easily compute $|\mathcal{W}|$ and $|\mathcal{F}|$, i.e. the signed area of the two polygons.

For example, for the wine polygon we have:

$$|\mathcal{W}| = \frac{1}{2} \sum_{i=1}^{S_W} x_i y_{i+1} - x_{i+1} y_i \ , \tag{1}$$

where $x_{S_W+1} = x_1$ and $y_{S_W+1} = y_1$.

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Notice that equation (1), can be interpreted as the cross-product of the 2 columns array (for example, wine case):

$$\begin{bmatrix} x_1 & & y_1 \\ x_2 & \ddots & y_2 \\ \vdots & \ddots & \vdots \\ x_{S_W} & & \\ x_{S_W+1} & \ddots & y_{S_W+1} \end{bmatrix}$$

Some maths

Now, the comparison will be made as follows.

- Determine the location of the centroids of the polygons.
- If necessary apply a roto-translation
- Compute the relative error

$$E(W, F) = \left| \frac{|\mathcal{W}| - |\mathcal{F}|}{|\mathcal{W}|} \right|$$

to check if the wine matches the food.

Back to our example

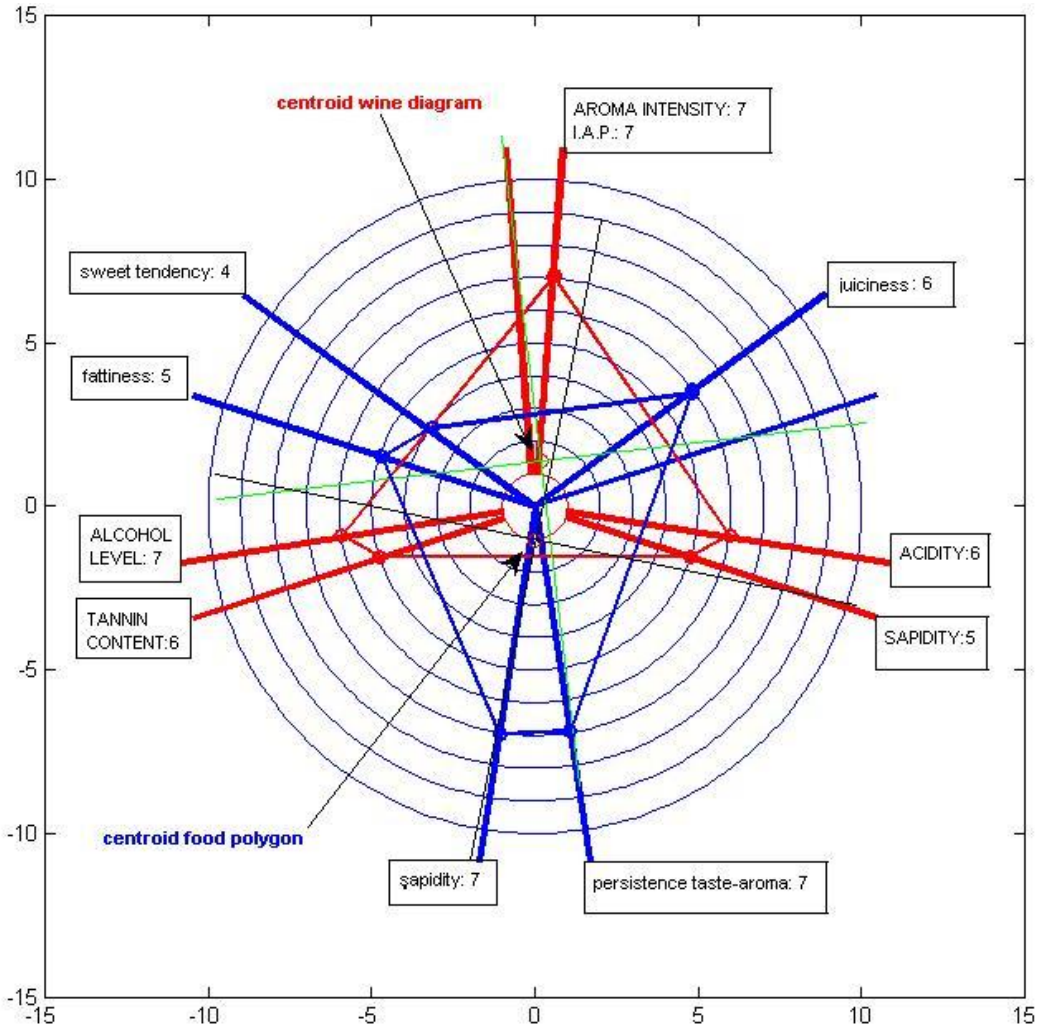
With the MATLAB function `polygeom` by H.J. Sommer, and the toolbox `Polygon clipper` by S. Hölz, both downloadable at `Matlab Central File Exchange` once we provide the vertices of the polygons we may compute:

- $|\mathcal{W}| \approx 54.1$ and $|\mathcal{F}| \approx 57.6$
- The centroid of the wine polygon is at $(0.2, 1.4)$ while that of the food is at $(0.01, -1.04)$.
- Perimeter of the wine polygon is 32.05 while that of the food is 32.37
- ... And more important, the two shapes mostly overlap (after a roto-translation)!

We may conclude that the ham and this wine are matching quite well.

For instance $E(W, F) \approx 6.5\%$

Again our example



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