3-dimensional Weakly Admissible Meshes

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Budapest, July 8, 2011

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Motivations and aims

(Weakly) Admissible meshes, (W)AM: play a central role in the construction of multivariate polynomial approximation processes on compact sets.
Motivations and aims

- **(Weakly) Admissible meshes, (W)AM**: play a central role in the construction of multivariate polynomial approximation processes on compact sets.

- **Theory vs computation**: 2-dimensional and (simple) 3-dimensional (W)AMs are easy to construct. What’s about more general domains such as (truncated) cones or rotational sets such as toroidal domains?
Main references


(Weakly) Admissible Meshes, (W)AM

Given a polynomial determining compact set $K \subset \mathbb{R}^d$.

**Definition**

An **Admissible Mesh** is a sequence of finite discrete subsets $A_n \subset K$ such that

$$
\|p\|_K \leq C \|p\|_{A_n}, \quad \forall p \in \mathbb{P}_n^d(K)
$$

(1)

holds for some $C > 0$ with $\text{card}(A_n) \geq N := \text{dim}(\mathbb{P}_n^d(K))$ that grows at most polynomially with $n$. 

A **Weakly Admissible Mesh**, or WAM, is a mesh for which the constant $C$ depends on $n$, i.e. $C = C(A_n)$, growing also polynomially with $n$. 

These sets and inequalities are also known as: $(L_\infty)$ discrete norming sets, Marcinkiewicz-Zygmund inequalities, stability inequalities (in more general functional settings).

**Optimal Admissible Meshes** the ones with $O(n^d)$ cardinality and can be constructed for some classes of compact sets (cf. [Kroó 2011], [Piazzon/Vianello 2010]).
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Admissible Meshes

In principle an AM of Markov compacts, i.e. \( K \subset \mathbb{R}^d \) s.t.

\[
\| \nabla p \|_K \leq M n^r \| p \|_K, \quad \forall p \in \mathbb{P}_n^d(K),
\]

where \( \| \nabla p \|_K = \max_{x \in K} \| \nabla p(x) \|_2 \)

Construction idea: take a uniform discretization of \( K \) with spacing \( \mathcal{O}(n^{-r}) \). The mesh will have cardinality of \( \mathcal{O}(n^{rd}) \) for real compacts or \( \mathcal{O}(n^{2rd}) \) for general complex domains.
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$r = 2$ for many (real convex) compacts: the construction and use of AM becomes difficult even for $d = 2, 3$ already for small degrees.

Too Big!!
Weakly Admissible Meshes: properties

**P1:** $C(\mathcal{A}_n)$ is invariant for affine transformations.

**P2:** any sequence of unisolvent interpolation sets whose Lebesgue constant grows at most polynomially with $n$ is a WAM, $C(\mathcal{A}_n)$ being the Lebesgue constant itself.

**P3:** any sequence of supersets of a WAM whose cardinalities grow polynomially with $n$ is a WAM with the same constant $C(\mathcal{A}_n)$.

**P4:** a finite union of WAMs is a WAM for the corresponding union of compacts, $C(\mathcal{A}_n)$ being the maximum of the corresponding constants.

**P5:** a finite cartesian product of WAMs is a WAM for the corresponding product of compacts, $C(\mathcal{A}_n)$ being the product of the corresponding constants.

**P7:** given a polynomial mapping $\pi_s$ of degree $s$, then $\pi_s(\mathcal{A}_{ns})$ is a WAM for $\pi_s(K)$ with constants $C(\mathcal{A}_{ns})$ (cf. [Bos et al. 2009]).
Weakly Admissible Meshes: properties

**P8:** any $K$ satisfying a Markov polynomial inequality like $\|\nabla p\|_K \leq Mn^r \|p\|_K$ has an AM with $O(n^{rd})$ points (cf. [Calvi/Levenberg 2008])

**P9:** The least-squares polynomial $L_{\mathcal{A}_n} f$ on a WAM is such that

$$\|f - L_{\mathcal{A}_n} f\|_K \lesssim C(\mathcal{A}_n) \sqrt{\text{card}(\mathcal{A}_n)} \min \{\|f - p\|_K, p \in \mathbb{P}_n^d(K)\}$$

**P10:** The Lebesgue constant of Fekete points extracted from a WAM can be bounded like $\Lambda_n \leq NC(\mathcal{A}_n)$

Moreover, their asymptotic distribution is the same of the continuum Fekete points, in the sense that the corresponding discrete probability measures converge weak-* to the pluripotential equilibrium measure of $K$ (cf. [Bos et al. 2009])
2-dimensional WAMS: disk, triangle, square

It was proved in [Bos et al. 2009] that, for the disk and the triangle there are WAMs with approximately $n^2$ points and the growth of $C(\mathcal{A}_n)$ is the same of an AM.
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- **Unit simplex**: starting from the WAM of the disk for polynomials of degree $2n$ containing only even powers, by the standard quadratic transformation 
  \[(u, v) \mapsto (x, y) = (u^2, v^2).\]
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(u, v) \mapsto (x, y) = (u^2, v^2).
$$

- **Square**: Chebyshev-Lobatto grid, Padua points.

**Notice**: by affine transformation these WAMs can be mapped to any other triangle ($P1$) or polygon ($P4$).
Polar symmetric WAMs for the disk

Figure: Symmetric polar WAMs for the disk: (Left) for degree $n = 11$ with $144 = (n + 1)^2$ points, (Right) for $n = 10$ with $121 = (n + 1)^2$ points.
WAMs for the quadrant and the triangle

Figure: A WAM of the first quadrant for polynomial degree $n = 16$ (left) and the corresponding WAM of the simplex for $n = 8$ (right).
2-dimensional WAMs

Optimal Lebesgue Gauss–Lobatto points on the triangle

A new set of optimal Lebesgue Gauss-Lobatto points on the simplex has recently been investigated by [Briani/Sommariva/Vianello 2011]. These points minimize the corresponding Lebesgue constant on the simplex, that grows like $O(n)$.

Figure: The optimal points for $n = 14$, cardinality $(n + 1)(n + 2)/2$.
Figure: A WAM for a quadrangular domain for $n = 7$ obtained by the bilinear transformation of the Chebyshev–Lobatto grid of the square $[-1, 1]^2$

$$\frac{1}{4}[(1-u)(1-v)A+(1+u)(1-v)B+(1+u)(1+v)C+(1-u)(1+v)D]$$
WAMs for (truncated) cones

Starting from a 2-dimensional domain WAM, we "repeat" the mesh along a Chebsyhev-Lobatto grid of the z-axis, as shown here in my handwritten notes (on my whiteboard).
Why these are WAMs?

From the previous picture

\[ |p(x, y, z)| \leq C(A_n) \|p\|_{A'(z)} \quad C(A_n) \equiv C(A'_n(z)) \]
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\[ |p(x, y, z)| \leq C(A_n) \|p\|_{A'_n(z)} \quad C(A_n) \equiv C(A'_n(z)) \]
\[ \|p\|_{A'_n(z)} = |p(\hat{x}_z, \hat{y}_z, z)| \text{ with } (\hat{x}_z, \hat{y}_z, z) \in A'_n(z) \]
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|p(x, y, z)| \leq C(A_n) \|p\|_{A'_n(z)} \quad C(A_n) \equiv C(A'_n(z))
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\|p\|_{A'_n(z)} = |p(\hat{x}_z, \hat{y}_z, z)| \text{ with } (\hat{x}_z, \hat{y}_z, z) \in A'_n(z)
\]

\[
\leq C(A_n) \|p\|_{\ell(\hat{\xi}_1, \hat{\xi}_2)} \text{ where } (\hat{\xi}_1, \hat{\xi}_2) \in A_n
\]

\[
\leq C(A_n) \max_{(x, y) \in A_n} \|p\|_{\ell(x,y)}
\]

\[
\leq O(C(A_n) \log n) \max_{(x, y) \in A_n} \|p\|_{\Gamma_n} = O(C(A_n) \log n) \|p\|_{B_n}
\]

where \(\Gamma_n\) are the Chebyshev-Lobatto points of \(l(x, y)\) and \(B_n = \bigcup_{(x,y) \in A_n} \Gamma_n(l(x,y))\).
Why these are WAMs?

From the previous picture

\[
|p(x, y, z)| \leq C(A_n)\|p\|_{A'_n(z)} \quad C(A_n) \equiv C(A'_n(z)) \\
\|p\|_{A'_n(z)} = |p(\hat{x}_z, \hat{y}_z, z)| \quad \text{with} \quad (\hat{x}_z, \hat{y}_z, z) \in A'_n(z) \\
\leq C(A_n)\|p\|_{\ell(\xi_1, \xi_2)} \quad \text{where} \quad (\hat{\xi}_1, \hat{\xi}_2) \in A_n \\
\leq C(A_n)\max_{(x, y) \in A_n}\|p\|_{\ell(x, y)} \\
\leq O(C(A_n)\log n)\max_{(x, y) \in A_n}\|p\|_{\Gamma_n} = O(C(A_n)\log n)\|p\|_{B_n}
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where \(\Gamma_n\) are the Chebyshev-Lobatto points of \(l(x, y)\) and 
\(B_n = \bigcup_{(x, y) \in A_n} \Gamma_n(\ell(x, y))\).

Cardinality.

\[
\#B_n = (n + 1)\#A_n - \#A_n + 1 = 1 + n\#A_n = O(n^3)
\]
WAMs for a cone

Figure: A WAM for the rectangular cone for $n = 7$

Here $C(A_n) = \mathcal{O}(\log^2 n)$ and the cardinality is $\mathcal{O}(n^3)$
A low dimension WAM for the cube

The cube can be considered as a *cylinder with square basis*. WAMs for the cube with dimension $O(n^3/4)$ were studied in [DeMarchi/Vianello/Xu 2009] in the framework of cubature and hyperinterpolation.

A WAM for the cube that for $n$ even has $(n+2)^3/4$ points and for $n$ odd $(n+1)(n+2)(n+3)/4$ points, is show here for a parallelepiped with $n = 4$ (here $\#A_n = 54$)

![Diagram of a low dimension WAM for the cube](image-url)
WAMs for a pyramid

Figure: A WAM for a non-rectangular pyramid and a truncated one, made by using Padua points for $n = 10$. Notice the generating curve of Padua points that becomes a spiral.

In this case $C(A_n) = \mathcal{O}(\log^2 n)$ and the cardinality is $\mathcal{O}(n^3/2)$.
WAMs for toroidal sections

Starting from a 2-dimensional WAM, $A_n$, by rotation around a vertical axis sampled at the $2n + 1$ Chabyshev-Lobatto points of the arc of circumference, we get WAMs for the torus, sections of the torus and in general toroids. The resulting cardinality will be $(2n + 1) \times \#A_n$.
Why these are WAMs?

From the previous ”picture" Given a polynomial \( p(x, y, z) \in \mathbb{P}^3_n \) we can write it in cylindrical coordinates getting

\[
p(x, y, z) = q(r, z, \phi) = s(x', y', \phi) \in \mathbb{P}^{2,(x',y')}_n \otimes T^\phi_n
\]

since

\[
x^i y^j x^k = (r \cos \phi)^i (r \sin \phi)^j z^k (r_0 + x')^i \cos^i \phi (r_0 + y')^j \sin^j \phi (r_0 + y')^k
\]
In this case \( C(A_n) = \mathcal{O}(\log^2 n) \) and the cardinality is \( \mathcal{O}(2n^3) \).
WAMs for toroidal sections: Padua points

Figure: Padua points on the toroidal section with $z_0 = 0$, $r_0 = 3$ and opening $-2/3\pi \leq \theta \leq 2/3\pi$.

In this case $C(A_n) = \mathcal{O}(\log^2 n)$ and the cardinality is $\mathcal{O}(n^3)$. 
WAMs for toroidal sections: simplex, GLL points

Figure: GLL points for $n = 7$ on the torus section

Cardinality is $O(n^3)$
WAMs for toroidal sections: equilateral triangle, GLL points

Figure: GLL points for $n = 7$ on the torus section for an equilateral triangle
Some notation

- Let $A_n$ be an AM or WAM of $K \subset \mathbb{R}^d$ (or $\mathbb{C}^d$)
Some notation

- Let $\mathcal{A}_n$ be an AM or WAM of $K \subset \mathbb{R}^d$ (or $\mathbb{C}^d$)
- The rectangular Vandermonde-like matrix

$$V(a; p) = V(a_1, \ldots, a_M; p_1, \ldots, p_N) = [p_j(a_i)] \in \mathbb{C}^{M \times N}, \quad M \geq N$$

where $a = (a_i)$ is the array of the points of $\mathcal{A}_n$ and $p = (p_j)$ the basis of $\mathbb{P}_n^d$. 
A greedy maximization of submatrix volumes, implemented by the QR factorization with column pivoting of $V(a; p)^t$ gives the so-called Approximate Fekete points [Sommariva/Vianello 2009].
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A greedy maximization of nested square submatrix determinants, implemented by the LU factorization with row pivoting of $V(a;p)$ gives the so-called Discrete Leja points ([Bos/DeMarchi/et al. 2010] and already observed in [Schaback/De Marchi 2009]).
Consider the *square* Vandermonde matrix

\[
V = V(\xi, p) = (P_0 V_0)_{1 \leq i, j \leq N} := LU
\]

where \( V_0 = V(a, p) \), \( L = (L_0)_{1 \leq i, j \leq N} \) and \( U = U_0 \).
Consider the square Vandermonde matrix

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The polynomial interpolating a function \( f \) at \( \xi \), \( f = f(\xi) \in \mathbb{C}^N \) is

\[ L_n f(x) = c^t p(x) = (V^{-1} f)^t p(x) = (U^{-1} L^{-1} f)^t p(x) = d^t \phi(x) \quad (2) \]

where \( d^t = (L^{-1} f)^t \), \( \phi(x) = U^{-t} p(x) \).
Remarks

- Formula (2) is indeed a Newton-type interpolant. Since $U^{-t}$ is lower triangular, the basis $\phi$ is s.t.

$$\text{span}\{\phi_1, \ldots, \phi_N\} = \mathbb{P}_d^s, \quad 0 \leq s \leq n$$
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\[
\text{span}\{\phi_1, \ldots, \phi_{N_s}\} = \mathbb{P}_s^d, \quad 0 \leq s \leq n
\]

- \( V(\xi; \phi) = V(\xi; p)U^{-1} = LUU^{-1} = L \)

Hence, \( \phi_j(\xi_j) = 1 \) and \( \phi_j(x_i) = 0, \; i = 1, \ldots, j - 1, \) when \( j > 1. \)
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- Case $d = 1$. Since $\phi_j \in \mathbb{P}^1_{j-1}$, then 
  $\phi_j(x) = \alpha_j(x - x_1) \cdots (x - x_{j-1}), \ 2 \leq j \leq N = n + 1$ with
  $\alpha_j = ((x_j - x_1) \cdots (x_j - x_{j-1}))^{-1}$, i.e. the classical Newton basis with $d_j$ the classical divided differences up to $1/\alpha_j$. 

The connection between LU factorization and Newton Interpolation was recognized by [de Boor 2004] and in a more general way by [R. Schaback et al. 2008, 2009].
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Conic sections: disk

$K$ is the cone. Given an $n$, then

- The AFP are extracted from a WAM having $O(n^3)$ points
- The polynomial basis is the tensor product Chebyshev polynomial basis.
- The Lebesgue constant and the interpolation error has been computed on a mesh of control points (consisting of the original WAM with $2n$ instead of $n$).
Conic sections: disk

\( K \) is the cone. Given an \( n \), then

- The AFP are extracted from a WAM having \( \mathcal{O}(n^3) \) points
- The polynomial basis is the tensor product Chebyshev polynomial basis.
- The Lebesgue constant and the interpolation error has been computed on a mesh of control points (consisting of the original WAM with \( 2n \) instead of \( n \)).

We also computed the

1. least-square operator norm, \( \|L_{A_n}\| = \max_{x \in K} \sum_{i=1}^{M} |g_i(x)| \) where \( g_i, \ i = 1, \ldots, M \) are a set of generators and \( M \geq N = \dim \mathbb{P}_n^3 \) (cf. [Bos/De Marchi et al. 2010])

2. interpolation error \( \|f - p_n(f)\|_\infty \)

3. least-square error \( \|f - L_{A_n}(f)\|_\infty \)
Runge function on the cone

Lebesgue constant for a conical mesh

Degree vs. Lebesgue constant

Least-squares norm for a conical mesh

Degree vs. Least-squares norm

Interpolation abs err on AFP for the Runge function

Degree vs. Interpolation abs err

Least-squares abs err on AFP up to degree 20

Degree vs. Least-squares abs err
Numerical results

Cosine function on the rectangular cylinder

Notice: for polynomial interpolation on the cylinder a more stable basis is the Wade’s basis [Wade 2010, De Marchi/Marchioro/Sommariva 2010].

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Toric sections: disk, square

$K$ is now a toric section. Given $n$ then

- The AFP are extracted from a WAM having $(n + 1)^2(2n + 1)$ points in the case of the disk and $\frac{(n+1)(n+2)}{2}(2n + 1)$ in the case of the square (by using Padua points).
- The polynomial basis is the tensor product Chebyshev polynomial basis.
- The Lebesgue constant and the interpolation error has been computed on a mesh of control points (the original WAM of degree $2n$).

We computed as before least-square operator sup-norm, interpolation error $\|f - p_n(f)\|_\infty$ and least-square error $\|f - L_{A_n}(f)\|_\infty$. 
Runge function on the toric section

Lebesgue constant for a toroidal mesh

Least-squares norm for a toroidal mesh

Interp. abserr: Runge function on toric section

Least-squares abserr
Cosine function on the toric section using Padua points
Future works

- investigate other (general) domains
- correct polynomial basis for the domains
- improve the cputime for extraction of AFP and DLP
- applications: cubature, pdes, graphics and more
- RBF setting?
- ...
Future works

Dolomites Research Week on Approximation 2011, Alba di Canazei
5-9 September 2011

Dolomites Workshop on Constructive Approximation and Applications 2012, Alba di Canazei 7-12(?) September 2012
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Thank you for your attention