Master degree course on
Approximation Theory and Applications,

Lab exercises

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1. Find the optimal shape parameter ϵ_{opt} by using the *trial & error* approach for the following univariate functions

   (a)
   $$f_1(x) = \texttt{sinc}(x) = \frac{\sin \pi x}{\pi x} .$$

   (b)
   $$f_2(x) = \frac{3}{4}\left(\mathtt{e}^{-(9x-2)^2/4} + \mathtt{e}^{-(9x+1)^2/49}\right) + \frac{1}{2}\mathtt{e}^{-(9x-7)^2/4} - \frac{1}{10}\mathtt{e}^{-(9x-4)^2} ,$$
   è una *a variant* of the classical Franke function;

   (c)
   $$f_3(x) = (1 - |x - 0.5|)^5 \left(1 + 5|x - 0.5| - 27(x - 0.5)^2\right) ,$$
   this is an oscillatory $\mathcal{C}^2$ compactly supported RBF known as *Gneiting function* (here centered in $x = 0.5$).

   For each f_i, $i = 1, 2, 3$ create a table of the form

N	$\|P_{f_i} - f_i\|_\infty$	ϵ_{opt}
3		
5		
9		
17		
33		
65		

   where for each N, ϵ_{opt} is the point of minimum of the error curves (in the ∞-norm) varying $\epsilon \in [0, 20]$. As radial function for constructing the interpolant, consider the gaussian.

2. The second approach makes use of the *power function* (PF).

   (a) Make the plot of the sup-norm of the PF, $\|P_{\Phi,X}\|_\infty$, by using the formula
   $$P_{\Phi,X}(x) = \sqrt{\Phi(x, x) - (b(x))^t A^{-1}\, b(x)}$$
   with A the usual collocation matrix $A_{i,j} = \Phi(x_i, x_j)$ and $b(x) = [\Phi(\cdot, x_1), \cdots, \Phi(\cdot, x_N)]^t$, by varying $\epsilon \in [0, 20]$, again using the gaussian univariate kernel for $N = 3, 5, 9, 17, 33, 65$. This time we do the experiments in 2-dimension for $N = 9, 25, 81, 289$. Both as centers and evaluation points, take equsipaced points. As a proof that the method works, check that by increasing the number of centers, the maximum of the PF will decrease.
   Use the M-function `Powerfunction2D.m`.

(b) Make a table similar to that one of the previous exercise, but instead of the error column use the values cond(A) corresponding to ϵ_{opt}.

3. Apply the *cross-validation* or *Leave-One-Out method* on the following functions

   (a) The modified 1d Franke function

$$f(x) = \frac{3}{4}\left(\mathrm{e}^{-(9x-2)^2/4} + \mathrm{e}^{-(9x+1)^2/49}\right) + \frac{1}{2}\mathrm{e}^{-(9x-7)^2/4} - \frac{1}{10}\mathrm{e}^{-(9x-4)^2},$$

   by using the $\mathcal{C}^2$ Wendland radial function $\varphi_{3,1}(r) = (1-r)_+^4(4r+1)$ (use `DistanceMatrixCSRBF_new.m`), on equispaced and Chebyshev points. Make the plot of the error curves for $\epsilon \in [0, 20]$ and $N = 3, \ldots, 65$, as in the Table of exercise 1.

   (b) As in (a), this time for the 2-dimensional function

$$f(x, y) = \mathtt{sinc}(x)\mathtt{sinc}(y)$$

   by using the gaussian kernel.

The script `LOOCV2D.m` implements the *Leave-One-Out Cross Validation method*, in 2D.

The Matlab functions can be dowloaded at the link

http://www.math.unipd.it/∼demarchi/TAA2010