Master degree course on Approximation Theory and Applications,

Lab exercises

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1. • Write a Matlab code that construct the Bernstein approximant

$$B_n(f)(x) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right)\binom{n}{k} x^k (1-x)^{n-k} \tag{1}$$

   of the function $f(x) = x(1-x),\ x \in [0,1]$ for any degree n. Then show that

$$\lim_{n\to\infty} B_n(f) = f\,.$$

   Hints. For any n take the equispaced point set `x=linspace(0,1,n)`. As *evaluation points* consider a finer set, say `y=linspace(0,1,100)`. Then construct the matrix B of dimension $n \times 100$ whose colums correspond to the Bernstein polynomial at the generic y_j, that is $\binom{n}{k} y_j^k (1-y_j)^{n-k}$. Hence (1) is the scalar product ....

   • Try the same algorithm for the function $f(x) = x^2 - 3x + 1$.

   In both cases, compute for any n, the sup errors $e_n = \|f - B_n(f)\|_\infty$.

2. Compute the Lebesgue constants of Chebyshev and Chebyshev-Lobatto points (of $[-1,1]$) for $n = 3, \ldots, 50$.

   For the construction of the elementary Lagrange polynomials and the corresponding matrix on a set of evaluation points, please use the functions of Chapter 2 of the LectureNotes.