Master degree course on
Approximation Theory and Applications,
Lab exercises

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1. Using the **chebfun** *http://www.chebfun.org/examples/approx/BestApprox.html* code, `remez.m`, compute the BPA of $f(x) = x^{k+1}$ on $[-1, 1]$ for $k = 2, 4, 6$. As degree of the polynomial of best approximation take $n \leq k$.

   Repeat for the function $f(x) = \sin((2k + 1)x)$. What do we see?

```
%-----------------------------------------------------------------------------
function [p,err] = remez(f,n);  % compute deg n BA to chebfun f
iter = 1; delta = 1; deltamin = delta;
[a,b] = domain(f);
xk = chebpts(n+2); xo = xk;    % initial reference
sigma = (-1).^[0:n+1];          % alternating signs
normf = norm(f);
while (delta/normf > 1e-14) & iter <= 20
fk = feval(f,xk);                       % function values
w = bary_weights(xk);           % compute barycentric weights
h = (w*fk)/(w*sigma);         % levelled reference error
if h==0, h = 1e-19; end         % perturb error if necessary
pk = fk - h*sigma;                 % polynomial vals in the
p=chebfun(@(x)bary(x,pk,xk,w),n+1); % reference chebfun of trial
e = f - p;                               % polynomial chebfun of the
[xk,err] = exchange(xk,e,h,2); % error replace reference
if err/normf > 1e5                 % if overshoot, recompute with
[xk,err] = exchange(xo,e,h,1);% one-point exchange
end
xo = xk;
delta = err - abs(h);              % stopping value
if delta < deltamin,                % store poly with minimal norm
deltamin = delta;
pmin = p; errmin = err;
end
iter = iter + 1;
end
p = pmin; err = errmin;
%----------------------------------------------------------------------------------
```

2. **Leja sequences**

   Definition 1. *On $[a, b]$ take the point x_1. The point $x_s \in [a, b]$, $s = 2, 3, \ldots, N$ s.t.*

$$\prod_{k=1}^{s-1} |x_s - x_k| = \max_{x\in[a,b]} \prod_{k=1}^{s-1} |x - x_k| . \tag{1}$$

   is called a `Leja sequence` *for $[a, b]$.*

Write a Matlab code that constructs the sequence and then use it for constructing the interpolating polynomial in Newton form of the function $f(x) = \sin(x) + \cos(x),\ x \in [0, \pi]$.