Approximation Theory and Applications, 2015/16

First written test

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The candidate should write on **all** sheets surname, name and "registration number". Sheets are supplied by the teacher.
Do not use any notes and/or books.

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1. Give the definition of **unisovent set of points**.

   In $\mathbb{R}^2$ we have presented the construction of a set of points that satisfies the sufficient condition of unisolvency. Which set is and how is constructed?

2. What is the *arc-cosine metric*? Is this metric generalizable to higher dimensions?

3. There is a *greedy method* (or algorithm) that allows to generate points equispaced in a given metric μ. Describe it and give a couple of examples of point sets that can be generated by such an algorithm.

4. Prove the following theorem

   "Suppose $\Phi : \mathbb{R}^d \longrightarrow \mathbb{R}$ is continuous. Then Φ is positive definite if and only if Φ is even and we have, for all $N \in \mathbb{N}$ and all $c \in \mathbb{R}\backslash\{0\}$ and all pairwise distinct $\mathbf{x}_1, \ldots, \mathbf{x}_N$, we have $\displaystyle\sum_{i=1}^{N}\sum_{j=1}^{N} c_i c_j \Phi(\mathbf{x}_i - \mathbf{x}_j) > 0\,.$".

   Provide some examples of positive definite functions, explaining why these functions are PD.

5. Suppose Φ is CPD kernel of order 1 with the property $\Phi(0) \le 0$. Then the matrix A, with $A_{i,j} = \Phi(\mathbf{x}_i - \mathbf{x}_j)$, is invertible. Why?

6. Describe the Backus-Gilbert method for quasi-interpolation. In particular describe why this method is a Moving Least Squares approximation that reproduces polynomials of degree $\le d$ in s variables.

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Time: 2 hours.