Master degree course on Approximation Theory and Applications,

Lab exercises

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1. Plot the most important CPD radial basis functions

   - *Generalized multiquadrics*

     $$\Phi(\mathbf{x}) = (1 + \|\mathbf{x}\|^2)^\beta, \ \ \mathbf{x} \in \mathbb{R}^s, \ \beta \in \mathbb{R} \backslash \mathbb{N}_0$$

     with $\beta = 1/2$ (Hardy) we get SCDP of order 1 while with $\beta = 5/2$ we get a SCDP of order 3.

   - *Power functions* (shape parameter free!)

     $$\Phi(\mathbf{x}) = \|\mathbf{x}\|^\beta, \ \ \mathbf{x} \in \mathbb{R}^s, 0 < \beta \notin 2\mathbb{N}$$

     For $\beta = 3$ we get a SCPD of order 2 while for $\beta = 5$ a SCPD of order 3.

   - *thin-plate splines* (shape parameter free!)

     $$\Phi(\mathbf{x}) = \|\mathbf{x}\|^{2\beta} \log \|\mathbf{x}\|, \ \ \mathbf{x} \in \mathbb{R}^s, \beta \in \mathbb{N}$$

     The "classic" one is for $\beta = 1$ (SCPD of order 2), while for $\beta = 2$ we get a function SCPD of order 3. Verify that also these functions are *shape parameter free.*

2. Plot the **Wendland** functions $\varphi_{3,k}, \ k = 0, 1, 2, 3$ in the univariate and bivariate case

k	$\varphi_{3,k}$
0	$(1-r)_+^2$
1	$(1-r)_+^4(4r+1)$
2	$(1-r)_+^6(35r^2+18r+3)$
3	$(1-r)_+^8(32r^3+25r^2+8r+1)$

3. In $[0, 1]^2$ interpolate the Franke function by using the Wendland function, W2, $\varphi_{3,1}(r) = (1-r)_+^4(4r+1)$ on equispaced grids with

   $$N = 3^2, 5^2, 9^2, 17^2, 33^2, 65^2$$

   points in the two cases.

   - *Stationary case.* Consider $\epsilon = 0.7$. By passing to finer grids, duplicate ϵ (2ϵ) (which is equivalent to average the fill-distance).
   - *Nonstationary case.* The shape parameter $\epsilon = 0.7$ is kept fixed while N varies.

   Compute in both cases the RMSE.

   We will see that in the non-stationary case the "collocation" matrices are more and more full making the computations harder. For the solution of the linear system use the Matlab function `pcg.m` (pre-conditioned conjugate gradient).

   Note. Use the script `RBFInterpolation2DCSRBF.m` by making the appropriate changes. The script calls `kd_rangequery3.m` and `DistanceMatrixCSRBF_new.m`.

4. Repeat the computations with the basis W4 function $\varphi(r) = (1-r)_+^6(3+18r+3r^2-192r^3)$.
5. Plot the *power function* in 2D by using the *gaussian kernel* with $\epsilon = 6$ taking $N = 9^2 = 81$ equispaced points, Chebyshev and Halton. We will notice that the power function does depend on the choice of the points. Observe how changes its maximum value as N increases. Use the script `Powerfunction2D.m`.

The Matlab functions can be dowloaded at the link
http://www.math.unipd.it/∼demarchi/TAA2010