Master degree course on Approximation Theory and Applications,

Lab exercises

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For the today's exercises we need the following tools and some of the Matlab files downloadable at the link
http://www.math.unipd.it/~demarchi/TAA2010.

- *Halton sequence*: low-discrepancy set of points for the hypercube $[0,1]^s, \;\; s \geq 1$.
- *Fill-distance* (or *mesh size*) $h_{X,\Omega}$ of a set of points $X \subset \Omega$ and $\Omega \subseteq \mathbb{R}^s$.

$$h_{X,\Omega} = \sup_{x\in\Omega} \min_{\mathbf{x}_j \in X} \|x - x_j\|_2 \,, \tag{1}$$

- *Root Mean Square Error (RMSE)* between a function f and its interpolant P_f evaluated on a set of M distinct points in any dimension

$$RMSE := \frac{\|f - P_f\|_2}{\sqrt{M}} \,. \tag{2}$$

- We also consider the functions

$$f_s(\mathbf{x}) \;=\; 4^s \prod_{k=1}^{s} x_k(1-x_k), \quad \mathbf{x} = (x_1,\ldots,x_s) \in [0,1]^s \tag{3}$$

$$\mathtt{sinc}(\mathbf{x}) \;=\; \prod_{k=1}^{s} \frac{\sin(\pi x_k)}{\pi x_k} \,. \tag{4}$$

1 Proposed exercises

1. By using the built-in function `haltonset.m`, determine the Halton points for dimensions $s = 1, 2, 3$. Extract $100, 200$ and 500 points, and for any such set, compute the corresponding `fill-distance`, $h_{X,\Omega}$.

   To this aim, $h_{X,\Omega}$ can be determined with the Matlab command `hX=max(min(DME'))`, where `DME` is the *distance matrix* constructed by using `DistanceMatrix.m` on an evaluation set of points (for instance a finer equispaced grid of the data-sites `X`).

2. Show graphically the nested property of the Halton sequence,

$$H_{s,M} \subset H_{s,N}, \;\; M < N \,. \tag{5}$$

3. By using the function `DistanceMatrix.m` on Halton points for $s = 2$ compute the its condition number `cond`. What do you see?

4. Again for $s = 2$, but using the function `DistanceMatrixFit.m`, construct the RBF interpolant with basis $\phi_k(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}_k\|_2$ (i.e the translates at $\mathbf{x}_k$ of the basic function $\varphi(r) = r$), of the functions (3) e (4). Compute also the RMSE. How does the RMSE behave on changing the evaluation set?

5. Repeat the exercise for the gaussian $\varphi(r) = \mathrm{e}^{-\epsilon^2 r^2}$, $\epsilon > 0$ again for $s = 2$. For this exercise use the function `RBFInterpolation2D.m` which generalizes `DistanceMatrixFit.m`.