Master degree course on
Approximation Theory and Applications,

Lab exercises on

Estimating the "optimal" shape parameter

Prof. Stefano De Marchi

November 22, 2016

1. Find the optimal shape parameter ϵ_{opt} by using the *trial & error* approach for the following univariate functions

   (a)
   $$f_1(x) = \mathtt{sinc}(x) = \frac{\sin \pi x}{\pi x}\,.$$

   (b)
   $$f_2(x) = \frac{3}{4}\left(\mathrm{e}^{-(9x-2)^2/4} + \mathrm{e}^{-(9x+1)^2/49}\right) + \frac{1}{2}\mathrm{e}^{-(9x-7)^2/4} - \frac{1}{10}\mathrm{e}^{-(9x-4)^2}\,,$$
   is *a variant* of the classical Franke function;

   (c)
   $$f_3(x) = (1 - |x-0.5|)^5\left(1 + 5|x-0.5| - 27(x-0.5)^2\right)\,,$$
   this is an oscillatory $\mathcal{C}^2$ compactly supported RBF known as *Gneiting function* (here centered in $x = 0.5$).

   For each f_i, $i = 1, 2, 3$ create a table of the form

N	$\|P_{f_i} - f_i\|_\infty$	ϵ_{opt}
3		
5		
9		
17		
33		
65		

   where for each fixed N, ϵ_{opt} is the point of minimum of the error curves (in the ∞-norm) varying $\epsilon \in [0, 10]$. As radial function for constructing the interpolant, consider the Gaussian basic function.

2. The second approach makes use of the *power function* (PF), $P_{\Phi,X}$.

   (a) By using the formula $P_{\Phi,X}(x) = \sqrt{\Phi(x,x) - (b(x))^t A^{-1}\, b(x)}$ with A the usual collocation matrix $A_{i,j} = \Phi(x_i, x_j)$ and $b(x) = [\Phi(\cdot, x_1), \cdots, \Phi(\cdot, x_N)]^t$, plot $\|P_{\Phi,X}\|_\infty$ by varying $\epsilon \in [0, 10]$, using the bivariate gaussian kernel

   this time we do the experiments in 2-dimension for N = 9, 25, 81, 100. Both as centers and evaluation points, take equsipaced points. As a check that the method works, by increasing the number of centers the maximum of the PF will decrease.

   Use the purpouse the M-function `Powerfunction2D.m`.

(b) Make a table similar to that one of the previous exercise, but instead of the error column use the values `cond`(A) corresponding to ϵ_{opt}.

3. Apply the *Leave-One-Out Method Cross-Validation (LOOCV)* approach on the following functions

(a) The modified 1d Franke function

$$f(x) = \frac{3}{4}\left(\mathsf{e}^{-(9x-2)^2/4} + \mathsf{e}^{-(9x+1)^2/49}\right) + \frac{1}{2}\mathsf{e}^{-(9x-7)^2/4} - \frac{1}{10}\mathsf{e}^{-(9x-4)^2},$$

by using the $\mathcal{C}^2$ Wendland radial function $\varphi_{3,1}(r) = (1-r)_+^4(4r+1)$ (use `DistanceMatrixCSRBF_new.m`), on equispaced and Chebyshev points. Make the plot of the error curves for $\epsilon \in [0, 20]$ and $N = 3, \ldots, 65$, as in the Table of exercise 1.

(b) As in (a), this time for the 2-dimensional function

$$f(x, y) = \mathtt{sinc}(x)\mathtt{sinc}(y)$$

by using the gaussian kernel.

The corresponding script in 2D is `LOOCV2D.m`.

The Matlab functions can be dowloaded at the link
http://www.math.unipd.it/∼demarchi/TAA2010