Hermite-Birkhoff interpolation on scattered data in Banach spaces by cardinal basis operators

Giampietro Allasia and Cesare Bracco
Department of Mathematics "G. Peano"- University of Turin
giampietro.allasia, cesare.bracco@unito.it

Abstract

We consider a generalization to Banach spaces of Hermite-Birkhoff interpolation on arbitrarily distributed data, where the interpolation conditions involve Fréchet derivatives. The problem is not only interesting in itself, but it is also relevant since it allows to introduce interpolants with approximation order higher than in Lagrange interpolation. After giving a constructive characterization of a class of cardinal basis functions in Banach spaces, the interpolation problem is solved by a suitable combination of Taylor-Fréchet expansions and cardinal basis functions. The numerical performances of the interpolants can be furtherly improved by applying a localizing scheme and the corresponding approximation error is estimated. The theoretical results are supported by a noteworthy example in Hilbert spaces and numerical tests of the constructed methods.

References

[1] G. Allasia and C. Bracco (2011), Two interpolation operators on irregularly distributed data in inner product spaces, J. Comp. Appl. Math. **235** (7): 1763–1774.

[2] G. Allasia and C. Bracco (2011), Lagrange interpolation on arbitrarily distributed data in Banach spaces, Numer. Funct. Anal. Optim. **32** (2): 111–125.

[3] V.L. Makarov and V.V. Khlobystov, *Polynomial interpolation of operators*, J. Math. Sci. **86** (4) (1997), 2459-2521.

[4] P.M. Prenter, *Lagrange and Hermite interpolation in Banach spaces*, J. Approx. Theory **4** (1971), 419-432.

[5] L.A. Yanovich, M.V. Ignatenko, *Hermite-Birkhoff interpolation operator polynomials in the space of smooth functions*, Dokl. Nat. Akad. Nauk Belarusi **53** (5) (2009), 15-21.