1st Dolomites Workshop on Constructive Approximation and Applications

dedicated to Walter Gautschi for his 50 years of professional activity

Alba di Canazei, Trento (Italy), September 8–12, 2006

timetable and abstracts

http://www.sci.univr.it/~dwcaa06
dwcaa06@sci.univr.it

Our sponsors

Department of Computer Science	Department of Economic Science	University of Verona	Faculty of Science	University of Padua
Department of Pure and Applied Mathematics	Istituto Nazionale Di Alta Matematica	Municipality of Canazei	Val di Fassa	Province of Trento
Veneto Region	Province of Verona	Province of Padua	Banca di Credito Cooperativo di Cartura	Cavit

Contents

Contributed talks 35

Timetable

Friday, September 8			
	Plenary Session	**Session A**	**Session B**
7.45–8.45	Registration		
8.45–9.00	Opening		
	Chairman: R. Schaback		
9.00–9.45	**C. Brezinski** *The professional life of Walter Gautschi*, 22		
9.45–10.30	**C. de Boor** *GC_n - sets*, 24		
10.30–11.00	coffee break		
11.00–11.35	**G.H. Golub** *Reconstruction of a Polygon from its Moments*, 48		
		Chairman: L. Bos	Chairman: C. Brezinski
11.40–12.05		**J. Prestin** *Exponentially localized polynomial frames on the unit interval and the Euclidean sphere*, 67	**J.–P. Berrut** *A formula for the error of finite sinc-interpolation over a fixed finite interval*, 38
12.05–12.30		**F. Filbir** *Polynomial approximation on the sphere*, 43	**A. Cuyt** *Rational Approximation Theory and Scientific Computing*, 41
12.30–12.55		**M. Goetz** *Constructive Extremal Problems related to Inverse Balayage*, 47	**J. Van Deun** *Exact rational minimax approximation and interpolation with prescribed poles*, 74
12.55–13.20		**V. Katkovnik** *Multidimensional local polynomial approximations with adaptive order and support*, 54	**B. Verdonk** *Structured linear systems in shape reconstruction from moments*, 75
13.20–15.00	lunch		
	Chairman: C. de Boor		
15.00–15.45	**M. Buhmann** *Radial basis function interpolation*, 23		
15.45–16.15	coffee break		
16.15–17.00	**R. Schaback** *Kernel methods*, 30		
18.00	Welcome reception at the Canazei City Hall		

Saturday, September 9 - morning			
	Plenary Session	**Session A**	**Session B**
	Chairman: Y. Xu		
8.45–9.30 9.30–10.15	**B. Bojanov** *Interpolation by bivariate polynomials*, 19 **L. Bos** *Near Optimal Points for Polynomial Interpolation in Several Variables*, 20		
10.15–10.45	coffee break		
10.45–11.20	**L.N. Trefethen** *Is Gauss quadrature better than Clenshaw-Curtis?*, 72		
		Chairman: G.H. Golub	Chairman: T. Sauer
11.25–11.50		**J.A.C. Weideman** *Explicit Error Formulas for Interpolatory Quadrature Rules for Rational Integrands*, 77	**A. Foi** *Adaptive-shape neighborhood orthogonal transforms in image processing*, 44
11.50–12.15		**S.E. Notaris** *Error Bounds for Gauss Type Quadrature Formulae of Analytic Function*, 65	**M. Fornasier** *Fast reconstruction algorithm for sparse multivariate and vector valued data*, 45
12.15–12.40		**C.H. Rohwer** *The Discrete Pulse Transform*, 69	**L. Gemignani** *Structured matrix methods for computations with orthogonal matrix functions*, 46
12.40–13.05		**D. Laurie** *Generation of Radau-Kronrod and Lobatto-Kronrod quadrature formulas*, 59	**L. Traversoni** *A Physical View on Quaternion Wavelets*, 71
13.05–15.00	lunch		

Saturday, September 9 - afternoon			
	Plenary Session	**Session A**	**Session B**
	Chairman: M. Buhmann		
15.00–15.45	**I.H. Sloan** *Radial basis functions and polynomials — a hybrid approximation for the sphere*, 31		
15.45–16.30	**Y. Xu** *A New Reconstruction Algorithm for Radon Data*, 34		
16.30–17.00	coffee break		
		Chairman: B. Bojanov	Chairman: S. Waldron
17.00–17.25		**E. Berdysheva** *The natural quasi-interpolants of Durrmeyer type operators*, 37	**G. Mantica** *Polynomial sampling of Fractal Measures and Fourier-Bessel Functions*, 62
17.25–17.50		**A. Kroó** *On density of homogeneous polynomials on star-like and convex surfaces*, 57	**M. Piñar** *On differential properties for multivariate orthogonal polynomials*, 66
17.50–18.15		**S. Kunis** *Interpolation of scattered data on the sphere by localized polynomials*, 58	**J. Rodal** *The structure relations and difference representations for orthogonal polynomials of hypergeometric type in two discrete variables*, 68
21.00	Wine tasting evening at the coffee breaks room		

Sunday, September 10	
9.00	Excursion (meeting point: Hotel Alpe, Alba di Canazei)

Monday, September 11 - morning			
	Plenary Session	**Session A**	**Session B**
	Chairman: I.H. Sloan		
8.45–9.30	**A. Iske** *Multiscale Flow Simulation by Adaptive Particle Methods*, 26		
9.30–10.15	**J. Levesley** *Stabilising the Lattice Boltzmann Method using Ehrenfests' Steps*, 27		
10.15–10.45	coffee break		
		Chairman: H. Wendland	Chairman: D. Laurie
10.45–11.10		**S. Hubbert** *Thin Plate Spline Interpolation on the Unit Interval*, 50	**F. Dell'Accio** *New embedded boundary type cubature formulas on the simplex*, 42
11.10–11.35		**V. Michel** *Optimally Localizing Approximate Identities on the 2-sphere — an Alternative Approach*, 64	**J. Keiner** *Fast evaluation of quadrature formulae on the sphere*, 56
11.35–12.00		**S. Serra-Capizzano** *Spectral behavior of compact and Cesaro non-Hermitian perturbations of Hermitian (structured) sequences*, 70	**P.C. Leopardi** *Monotonicity of Jacobi polynomials and positive quadrature on the sphere*, 60
12.00–12.25		**S. Waldron** *Multivariate Jacobi polynomials with singular weights and the Bernstein operator*, 76	**A. Mazzia** *Meshless methods and numerical integration rules with applications to axisymmetric geomechanical problems*, 63
12.25–14.00	lunch		

Monday, September 11 - afternoon			
	Plenary Session	**Session A**	**Session B**
	Chairman: J. Levesley		
15.00–15.45	**G. Fasshauer** *On Choosing "Optimal" Shape Parameters for RBF Approximation*, 25		
15.45–16.30	**L. Montefusco** *Numerical aspects in surface reconstruction with Radial Basis Functions*, 28		
16.30–17.00	coffee break		
		Chairman: M. Bozzini	Chairman: S. Serra-Capizzano
17.00–17.25		**T. Bosner** *Non-uniform Tension Splines*, 39	**M.R. Capobianco** *Some remarks on the numerical computation of integrals on unbounded interval*, 40
17.25–17.50		**G. Jaklič** *On determining the dimension of dimension of the bivariate spline space $S^1_n(\Delta)$*, 51	**T. Hasegawa** *A triple-adaptive quadrature method based on the combination of the Ninomiya and the FLR schemes*, 49
17.50–18.15		**T. Ueno** *New spline basis functions for sampling approximations*, 73	**E.A. Karatsuba** *On approximation of exponential sums in certain physical problems*, 52
20.30	Social dinner at Hotel Alpe		

Monday, September 11 - afternoon	
	Special parallel Session on Approximation Methods in Finance Chairman: G. Fasshauer
17.00–17.35	**E. Larsson** *Improved radial basis function methods for multi-dimensional option pricing*, 91
17.35–18.10	**B. Waterhouse** *Using lattice rules to solve high-dimensional integration problems from mathematical finance*, 92

Monday, September 11 - afternoon	
	Poster Session
14.00–15.00	**E. Al-Aidarous** *On generalized Lindelöf orthogonal polynomials with applications*, 81 **M. Caliari** *Bivariate Lagrange interpolation at the Padua points: computational aspects*, 82 **I. Caraus** *Approximate solution of singular integro-differential equations in Generalized Hölder spaces*, 83 **L. De Biase** *A very simple (but very effective) spline approximation of the Priestley Glacier*, 84 **C. Drioli** *On the use of Kernel-based methods in physical modeling of sounds*, 85 **G. Jaklič** *Three-pencil lattice in a closed form*, 86 **M.R. Russo** *An Approach by Vector Extrapolation Methods to the Gummel Map*, 87 **A. Sommariva** *Meshfree Cubature by Radial Basis Functions*, 88

Tuesday, September 12 - morning	
	Plenary Session
	Chairman: A. Iske
9.00–9.45	**M. Bozzini** *Polyharmonic B-splines: an approximation method for scattered data of extra-large size*, 21
9.45–10.30	**T. Sauer** *Geometric lattices: construction and error*, 29
10.30–11.00	coffee break
11.00–11.45	**H. Wendland** *Recent Results on Meshless Symmetric Collocation*, 32
11.45–12.00	Greetings

Invited talks

Interpolation by bivariate polynomials

B. Bojanov*
Departament of Mathematics and Informatics
University of Sofia (Bulgaria)

We discuss some recent results on interpolation by polynomials in two variables using the classical point value table as well as interpolation based on Radon projections.

*boris@fmi.uni-sofia.bg

Near Optimal Points for Polynomial Interpolation in Several Variables

L. Bos*
Department of Mathematics and Statistics
University of Calgary (Canada)

We discuss some nodal sets for Lagrange polynomial interpolation in several variables, including the recently introduced so-called Padua points for a square in $\mathbb{R}^2$ that have been shown to have optimal rate of growth of the Lebesgue constant. We also discuss some numerical applications. This includes joint work with M. Caliari, S. De Marchi, M. Vianello and Y. Xu.

*lpbos@math.ucalgary.ca

Polyharmonic B-splines: an approximation method for scattered data of extra-large size

M. Bozzini[*], M. Rossini[†]
Department of Mathematics and Applications
University of Milan-Bicocca (Italy)

L. Lenarduzzi[‡]
CNR IMATI, Milan (Italy)

Recently polyharmonic B-splines close to a gaussian were studied.

In this talk we present a fast method exploiting these functions, in order to recover surfaces from a very large sample of scattered data corrupted by noise and eventually with outliers. Some real examples will be shown.

[*] mira.bozzini@unimib.it
[†] milvia.rossini@unimib.it
[‡] licia@mi.imati.cnr.it

The professional life of Walter Gautschi

C. Brezinski*
Laboratory "Paul Painlevé"
University of Lille (France)

In this talk, I will review the most important results obtained by Walter Gautschi in the domains of ordinary differential equations, computation of special functions, interpolation, continued fractions, Padé approximation, convergence acceleration, Gauss-type quadratures, Féjer quadratures, Chebyshev-type quadratures, and orthogonal polynomials.

*claude.brezinski@univ-lille1.fr

Radial basis function interpolation

M. Buhmann*
Institute of Mathematics
Justus-Liebig-University, Giessen (Germany)

We consider radial basis function approximation by interpolation in any dimension. The existence and properties of the radial basis function interpolation depend not only on the choice of radial basis functions but also in some circumstances on the location of the data points. We will consider these aspects of radial basis functions especially for the celebrated multiquadric radial basis function and for the Gaussian kernels.

*martin.buhmann@math.uni-giessen.de

GC_n-sets

C. de Boor*
Department of Computer Sciences and Department of Mathematics
University of Wisconsin-Madison (USA)

A GC_n-set, as introduced by Chung and Yao in 1977, is a set X in R^d correct for interpolation from $\Pi_{\leq n}$ with the additional 'geometric condition' that, for each $x \in X$, the set $X \backslash x$ lies in the union of at most n hyperplanes. The talk will translate to R^d what is known about such sets in the plane, with special attention to the Gasca-Maeztu conjecture that, for $d = 2$, any GC_n set must have (at least one set of) $n + 1$ collinear points.

*deboor@cs.wisc.edu

On Choosing "Optimal" Shape Parameters for RBF Approximation

G. Fasshauer*
Department of Applied Mathematics
Illinois Institute of Technology, Chicago (USA)

Many radial basis functions contain a free parameter that can be tuned by the user in order to obtain a good balance between accuracy and stability. This dependence is known in the literature as the *uncertainty* or *trade-off* principle. The most popular strategy for choosing an "optimal" shape parameter is the leave-one-out cross validation algorithm proposed by Rippa [1] in the setting of scattered data interpolation.

We will report on extensions of this approach that can be applied in the setting of RBF pseudospectral methods for the solution of PDEs. Alternative strategies are investigated that include both the use of multiple shape parameters and more stable basis functions.

Bibliography

[1] S. Rippa, *An algorithm for selecting a good value for the parameter c in radial basis function interpolation*, Adv. Comput. Math. **11**, 193–210.

*fasshauer@iit.edu

Multiscale Flow Simulation by Adaptive Particle Methods

A. Iske*
Department of Mathematics
University of Hamburg (Germany)

Particle models have provided very flexible discretization schemes for the numerical simulation of multiscale phenomena in time-dependent evolution processes. This talk reports on recent advances concerning the design and analysis of adaptive particle methods for flow simulation. To this end, basic tools from approximation, including scattered data reconstruction by polyharmonis splines, are first discussed, before the construction of adaptive multiscale algorithms is explained, and selected of their computational aspects are addressed. The good performance of the resulting numerical algorithms is demonstrated in comparison with state-of-the-art simulation methods, where test case scenarios from real-world applications are utilized.

*iske@math.uni-hamburg.de

Stabilising the Lattice Boltzmann Method using Ehrenfests' Steps *

J. Levesley[†], R. Brownlee[‡], A. Gorban[§]
Department of Mathematics
University of Leicester (United Kingdom)

The lattice-Boltzmann method (LBM) and its variants have emerged as promising, computationally efficient and increasingly popular numerical methods for modelling complex fluid flow. However, it is acknowledged that the method can demonstrate numerical instabilities, e.g., in the vicinity of shocks. We propose a simple and novel technique to stabilise the lattice-Boltzmann method by monitoring the difference between microscopic and macroscopic entropy. Populations are returned to their equilibrium states if a threshold value is exceeded. We coin the name Ehrenfests' steps for this procedure in homage to the vehicle that we use to introduce the procedure, namely, the Ehrenfests' idea of coarse-graining.

The one-dimensional shock tube for a compressible isothermal fluid is a standard benchmark test for hydrodynamic codes. We observe that, of all the LBMs considered in the numerical experiment with the one-dimensional shock tube, only the method which includes Ehrenfests' steps is capable of suppressing spurious post-shock oscillations.

We can also compare our new method with smoothed particle hydrodynamic simulations, another of the commonly used simulation techniques for complex fluid flow.

*Work supported by EPSRC
[†] jl1@mcs.le.ac.uk
[‡] rab14@mcs.le.ac.uk
[§] ag153@mcs.le.ac.uk

Numerical aspects in surface reconstruction with Radial Basis Functions*

L.B. Montefusco†, G. Casciola‡, S. Morigi§
Department of Mathematics
University of Bologna (Italy)

The problem of reconstructing surfaces from scattered data using Radial Basis Functions (RBF) is a widely investigated inverse problem. Hence, a particular attention must be paid to the numerical aspects involved in its solution. In fact, it is well known that for very large and highly unevenly distributed sets of data points the matrices of the resulting linear systems can be very poorly conditioned and the instability grows as the regularity of the RBF increases. In this talk we will present some different strategies for circumventing this problem while still maintaining a good level of the reproducing quality of the reconstruction. A first proposal is concerning with a local approach to the reconstruction using a partition of unity strategy that allows us to decompose the large original data set into smaller subsets with a consequent reduction of numerical problems. This local control on the reconstructed surface let us adapt the reconstruction to the shape of the data [1]. Nevertheless, the intrinsic nature of the RBF can produce numerical instabilities even for small data sets if the data are unevenly distributed. To afford the latter problem we propose a *metric regularization* approach based on anisotropic RBF which can be very efficient in case of particular data distributions [2].

Bibliography

[1] G. Casciola, D. Lazzaro, L.B. Montefusco, and S. Morigi, *Fast surface reconstruction and hole filling using Radial Basis Functions*, Numer. Algorithms **39** (2005), 289–305.

[2] ———, *Shape preserving surface reconstruction using locally anisotropic RBF Interpolants*, Comput. Math. Appl. (2006), to appear.

*This work has been supported by PRIN 2004 and MIUR 2005 projects
†montelau@dm.unibo.it
‡casciola@dm.unibo.it
§morigi@dm.unibo.it

Geometric lattices: construction and error

T. Sauer*
Institute of Mathematics
Justus-Liebig-University, Giessen (Germany)

The explicit construction of point sets Ξ which allow for unique interpolation by Π_n, the polynomials of total degree at most n, is still an important problem in multivariate interpolation. Many such construction emerge from the *geometric condition* introduced in the now classical paper by Chung and Yao. This geometric condition corresponds to the algebraic property that all *Lagrange fundamental polynomials* ℓ_ξ, $\xi \in \Xi$, defined by $\ell_\xi(\xi') = \delta_{\xi,\xi'}$, $\xi, \xi' \in \Xi$, can be factorized into linear polynomials, a requirement that is *always* satisfied in the univariate case but seldom in several variables.

The topic of the talk is to point out that such configurations provide very simple formulas for the error $f - L_n f$ of the interpolation operator L_n applied to a sufficiently smooth function which are ruled by few *geometric* quantities and to introduce another method for the construction of such lattices which is, surprisingly, based on univariate Haar spaces.

This is joint work with Jesús Carnicer and Mariano Gasca.

*Tomas.Sauer@math.uni-giessen.de

Kernel methods

R. Schaback*
Institute for Numerical and Applied Mathematics
Georg-August-University, Göttingen (Germany)

Kernels are valuable tools in various fields of Numerical Analysis, including approximation, interpolation, meshless methods for solving partial dierential equations, neural networks, and Machine Learning. This contribution explains why and how kernels are applied in these disciplines. It uncovers the links between them, as far as they are related to kernel techniques. It addresses non-expert readers and focuses on practical guidelines for using kernels in applications.

*schaback@math.uni-goettingen.de

Radial basis functions and polynomials — a hybrid approximation for the sphere

I.H. Sloan*
School of Mathematics and Statistics
University of New South Wales, Sidney (Australia)

Many researchers have discussed approximation by radial basis functions on a sphere, using scattered data. Usually there is no polynomial component in such approximations if, as here, the kernel that generates the radial functions is (strictly) positive definite. On the other hand, the utility of polynomials for approximating slowly varying components is well known – an extreme case is the NASA model of the earth's gravitational potential, which represents the potential by a purely polynomial approximation of high degree. In this joint work with Alvise Sommariva we propose a hybrid approximation, in which there is a radial basis functions component to handle the rapidly varying and localised aspects, but also a polynomial component to handle the more slowly varying and global parts. The convergence theory (including a doubled rate of convergence for sufficiently smooth functions) makes use of the "native space" associated with the positive definite kernel (with no polynomial involvement in the definition). A numerical experiment for a simple model with a geophysical flavour establishes the potential value of the hybrid approach.

*i.sloan@unsw.edu.au

Recent Results on Meshless Symmetric Collocation

H. Wendland*
Institute for Numerical and Applied Mathematics
Georg-August-University, Göttingen (Germany)

Meshless collocation methods for the numerical solution of partial differential equations have recently become more and more popular. They provide a greater flexibility when it comes to adaptivity and time-dependent changes of the underlying region.

Radial basis functions or, more generally, (conditionally) positive definite kernels are one of the main stream methods in the field of meshless collocation. In this talk, I will give a survey of well-known and recent results on meshless, symmetric collocation for boundary value problems using positive definite kernels. In particular, I will address the following topics

1. Well-posedness of the problem, particularly for differential operators with non-constant coefficients.
2. Error analysis in Sobolev spaces.
3. Stability analysis of the collocation matrix.
4. Stabilization by smoothing.
5. Examples.

I will refer to the previous results in [1, 3, 2, 5]. However, this talk is mainly based upon recent results from joint work with Francis J. Narcowich and Joseph D. Ward from Texas A&M University, with Christian Rieger from the University of Göttingen, and with Peter Giesl from the Technical University of Munich [4, 6, 7, 8].

Bibliography

[1] G.E. Fasshauer, *Solving partial differential equations by collocation with radial basis functions*, Surface Fitting and Multiresolution Methods (A. L. Méhauté, C. Rabut, and L. L. Schumaker, eds.), Vanderbilt University Press, Nashville, 1997, pp. 131–138.

[2] C. Franke and R. Schaback, *Convergence order estimates of meshless collocation methods using radial basis functions*, Adv. Comput. Math. **8** (1998), 381–399.

[3] ———, *Solving partial differential equations by collocation using radial basis functions*, Appl. Math. Comput. **93** (1998), 73–82.

[4] P. Giesl and H. Wendland, *Meshless collocation: Error estimates with application to dynamical systems*, Preprint, 2006.

[5] R. Lorentz, F.J. Narcowich, and J.D. Ward, *Collocation discretization of the transport equation with radial basis functions*, Appl. Math. Comput. **145** (2003), 97–116.

*wendland@math.uni-goettingen.de

[6] F.J. Narcowich, J.D. Ward, and H. Wendland, *Sobolev bounds on functions with scattered zeros, with applications to radial basis function surface fitting*, Math. Comput. **74** (2005), 643–763.

[7] H. Wendland, *On the stability of meshless symmetric collocation for boundary value problems*, Preprint, 2006.

[8] H. Wendland and C. Rieger, *Approximate interpolation with applications to selecting smoothing parameters*, Numer. Math. **101** (2005), 643–662.

A New Reconstruction Algorithm for Radon Data

Y. Xu*
Department of Mathematics
University of Oregon (USA)

O. Tischenko†, C. Hoeschen‡
Institute of Radiation Protection
GSF-National Research Center
for Environment and Health
Neuherberg (Germany)

We discuss a new algorithm for reconstruction of images from Radon data. The algorithm is called OPED as it is based on Orthogonal Polynomial Expansion on the Disk. OPED is fundamentally different from the filtered back projection (FBP) method, the main algorithm currently being used in the computer tomography (CT) and medical image. OPED allows one to use fan geometry directly without the additional procedures such as interpolation or rebinning. It reconstructs high degree polynomials exactly and converges uniformly for smooth functions without the assumption that functions are band-limited. Our initial test indicates that the algorithm is stable, provides high resolution images, and has a small global error.

Bibliography

[1] Y. Xu, *A direct approach to the reconstruction of images from Radon projections*, Adv. in Applied Math. (2006), in print.

[2] Y. Xu, O. Tischenko, and C. Heoschen, *New tomographic reconstruction algorithm*, submitted.

[3] ______, *A new reconstruction algorithm for Radon data*, SPIE Proceedings of Medical Imaging (2006), in print.

*yuan@uoregon.edu
†oleg.tischenko@gsf.de
‡christoph.hoeschen@gsf.de

Contributed talks

The natural quasi-interpolants of Durrmeyer type operators

E. Berdysheva*
Institute for Applied Mathematics and Statistics
University of Hohenheim, Stuttgart (Germany)

We shall discuss approximation properties of the natural quasi-interpolants of Durrmeyer type operators such that the Bernstein-Durrmeyer operator on the d-dimensional simplex, the Szász-Mirakjan-Durrmeyer and the Baskakov-Durrmeyer operators. One of the main results in the background is the complete monotonicity property of the kernels of Durrmeyer operators which is proved on the base of representations of the kernels in terms of special functions. Partly joint work with K. Jetter and J. Stöckler.

*elena.berdysheva@uni-hohenheim.de

A formula for the error of finite sinc–interpolation over a fixed finite interval*

J.–P. Berrut†
Department of Mathematics
University of Fribourg (Switzerland)

Sinc–interpolation is an infinitely smooth interpolation on the whole real line based on a series of shifted and dilated sinus–cardinalis functions used as Lagrange basis. It often converges very rapidly, so for example for functions analytic in an open strip containing the real line and which decay fast enough at infinity. This decay does not need to be very rapid, however, as in Runge's function $1/(1 + x^2)$. Then one must truncate the series, and this truncation error is much larger than the discretisation error (it decreases algebraically while the latter does it exponentially).

In our talk we will give a formula for the error commited when merely using function values from a finite interval symmetric about the origin. The main part of the formula is a polynomial in the distance between the nodes whose coefficients contain derivatives of the function at the extremities.

*Work supported by the Swiss National Science Foundation
†`jean-paul.berrut@unifr.ch`

Non-uniform Tension Splines

T. Bosner[*], M. Rogina[†]
Department of Mathematics
University of Zagreb (Croatia)

We describe explicitly each stage of a numerically stable algorithm for calculating with tension B-splines with non-uniform choice of tension parameters. These splines are piecewisely in the kernel of $D^2(D^2 - p^2)$, defined on arbirary meshes, with a different choice of the tension parameter p on each interval. The algorithm provides values of the associated B-splines and their generalized and ordinary derivatives by performing positive linear combinations of positive quantities, described as lower-order tension splines. We show that nothing else but the knot insertion algorithm and good approximation of a few elementary functions is needed to achieve machine accuracy. The underlying theory is that of splines based on Chebyshev canonical systems which are not smooth enough to be ECC-systems. The continuity of the second generalized derivatives required by the Chebyshev theory conflicts with the classical C^2-smoothness for tension splines. Nevertheless, one can first construct Chebyshev tension spline with known jumps in the second derivative, and then use de Boor algorithm and quasi-Oslo type algorithms for evalution of classical non-uniform tension splines.

Bibliography

[1] V. Horvat and M. Rogina, *Tension spline collocation methods for singularly perturbed Volterra integro-differential and Volterra integral equations*, J. Comput. Appl. Math. **140**, 381–402.

[2] M. Marušić, *Stable calculation by splines in tension*, Grazer Mathematische Berichte **328**, 65–76.

[3] ———, *A fourth/second order accurate collocation method for singularly perturbed two-point boundary value problems using tension splines*, Numer. Math. **88** (2001), 135–158.

[4] M. Marušić and M. Rogina, *A collocation method for singularly perturbed two-point boundary value problems with splines in tension*, Adv. Comput. Math. Vol. **6** (1996), no. 1, 65–76.

[5] M. Rogina and T. Bosner, *On calculating with lower order Chebyshev splines*, Curves and Surfaces Design (P. J. Laurent, P. Sabloniere, and L. L. Schumaker, eds.), Vanderbilt Univ. Press, Nashville, 2000, pp. 343–353.

[6] ———, *A de Boor type algorithm for tension splines*, Curve and Surface Fitting (J.-L. A. Cohen, Merrien and L. L. Schumaker, eds.), Nashboro Press, Brentwood, 2003, pp. 343–352.

[*] tinab@math.hr
[†] rogina@math.hr

Some remarks on the numerical computation of integrals on unbounded interval

M.R. Capobianco*
CNR-IAC "Mauro Picone"
Naples (Italy)

G. Criscuolo†
Department of Mathematics and Applications
University of Naples "Federico II" (Italy)

We investigate numerical methods for the approximate evaluation of integrals on unbounded interval of the form

$$\int_0^\infty f(x)w_\alpha(x)dx,$$

where $w_\alpha(x) = x^\alpha e^{-x},\ \alpha > -1$, and f satisfies suitable smoothness conditions.

Consider the classical Gauss–Laguerre quadrature formula

$$\int_0^\infty f(x)w_\alpha(x)dx = \sum_{k=1}^{m} \lambda_{m,k} f(x_{m,k}) + R_m(w_\alpha f),$$

where $x_{m,1} < x_{m,2} < ... < x_{m,m}$. The availability of efficient mathematical software to compute this formula makes it advantageous. Nevertheless, since the coefficients $\lambda_{m,k}$ of the Gauss–Laguerre rule decay exponentially as $x_{m,k} \to \infty$, the practical computation may exhibit numerical cancellation when m is too large. Therefore, the practical use of the previous rule is useful only when f does not diverge too fast as $x \to \infty$.

In the following we consider essentially two cases. At the first, we assume that f diverges "rapidly" as $x \to \infty$. In this situation we assume the Gauss-Laguerre formula as an approximation of the integral $\int_0^{x_{m,m}} f(x)w_\alpha(x)dx$, observing that even if $\int_{x_{m,m}}^\infty f(x)w_\alpha(x)dx \to 0$ as $m \to \infty$, it cannot be neglected. In other words, in the practical computations the Gauss–Laguerre formula works only to approximate the integral $\int_0^{x_{m,m}} f(x)w_\alpha(x)dx$ when f diverges too fast. In this case we also compare the Gauss–Laguerre formula with a rule on equispaced knots on the interval $[0, x_{m,m}]$ and some "truncated" Gauss–Laguerre formulas.

Fortunately, in many applications, as for instance in the integral equations over $(0, \infty)$, we have to approximate integrals on unbounded interval when f has a particular known behaviour as $x \to \infty$. For the class of such functions not rapidly diverging, we prove that the Gauss–Laguerre quadrature rule and the related truncated rule converge with the same order. Nevertheless, in order to avoid computational problems, it is of interest to have a convergent quadrature rule using a number n of knots such that $n < O(m)$. We construct a quadrature rule as simple as the classical Gauss–Laguerre formula using $n < O(m)$ knots proving a convergence result. The numerical examples confirm the theoretical results.

*r.capobianco@na.iac.cnr.it
†giuliana.criscuolo@unina.it

Rational Approximation Theory and Scientific Computing

A. Cuyt*
Department of Mathematics and Computer Science
University of Antwerp (Belgium)

In recent years several highly technological problems could profit from some classical results in rational approximation theory, as can be seen from the existing literature. We discuss following selected problems:

1. The computation of the packet loss probability as a function of the buffer size in the context of multiplexing techniques, to support variable bit rate communication, can be realized in almost real-time making use of multipoint Padé-type approximants.

2. The reconstruction of general two- and three-dimensional shapes from indirect measurements such as bi- and trivariate moment information, is possible because of the relationship between several integral transforms and homogeneous multivariate Padé approximants.

3. Models describing complicated physical devices or extremely time-consuming simulations, can be highly simplified using adaptive scattered rational interpolation, while maintaining at the same time a required accuracy.

4. A large collection of special functions from science and engineering, can be evaluated reliably and efficiently by means of modified continued fraction approximants, guaranteeing evaluations up to a user defined accuracy which can be chosen from a few digits to several hundreds or thousands, truncation and round-off error included.

*annie.cuyt@ua.ac.be

New embedded boundary type cubature formulas on the simplex

F.A. Costabile*, **F. Dell'Accio**†
Department of Mathematics
University of Calabria (Italy)

In this talk we consider the problem of the approximation of the integral of a smooth enough function $f(x,y)$ on the simplex $\Delta_2 \subset \mathbb{R}^2$ by cubature rules of the form

$$\int_{\Delta_2} f(x,y)\,dxdy = \sum_{k=1}^{3} \sum_{i,j} A_{ij}^{k} \frac{\partial^{i+j}}{\partial x^i \partial y^j} f(x_k, y_k) + E(f)$$

where the nodes (x_k, y_k), $k = 1, 2, 3$ are the vertices of the simplex. Such kind of formulas belong to a more general class of formulas for numerical integration, which are called Boundary Type Quadrature Formulas (BTQF). We present two classes of such formulas that are exact for algebraic polynomials and generate embedded pairs. We give bounds for the truncation errors and conditions for convergence. Finally, we provide an algorithm for automatic computation and numerical examples.

*costabil@unical.it
†fdellacc@unical.it

Polynomial approximation on the sphere

F. Filbir[*]
Institute for Biomathematics and Biometry
GSF National Research Center
Neuherberg (Germany)

W. Themistoclakis[†]
CNR-IAC "Mauro Picone"
Naples (Italy)

We consider the problem of approximately reconstructing a function f defined on the surface of the unit sphere in the Euclidean space $\mathbb{R}^{q+1}$, using samples of f at scattered sites. A central role is played by the construction of a new operator for polynomial approximation, which is a uniformly bounded quasi–projection in the de la Vallée Poussin style, i.e. it reproduces spherical polynomials up to a certain degree and has uniformly bounded L^p operator norm for $1 \leq p \leq \infty$. Using certain positive quadrature rules for scattered sites due to Mhaskar, Narcowich and Ward, we discretize this operator obtaining a polynomial approximation of the target function which can be computed from scattered data and provides the same approximation degree of the best polynomial approximation. To establish the error estimates we use Marcinkiewicz-Zygmund inequalities, which we derive from our continuous approximating operator. For all the constants in the Marcinkiewicz-Zygmund inequalities as well as in the error estimates, we give concrete bounds.

[*] `filbir@gsf.de`
[†] `wt@na.iac.cnr.it`

Adaptive-shape neighborhood orthogonal transforms in image processing*

A. Foi†, V. Katkovnik‡, K. Egiazarian§
Institute of Signal Processing
Tampere University of Technology (Finland)

In the last decade, significant research has been made towards the development of region-oriented, or *shape-adaptive*, transforms. The main intention is to construct a system (frame, basis, etc.) that can efficiently be used for the analysis and synthesis of arbitrarily shaped image segments, where the data exhibit some uniform behavior. In this talk we give an overview of the most significant approaches which have appeared in this area (e.g. [2, 4, 3]), highlighting their pecularities and advantages. Our illustration concerns with image processing problems, as one of the most competitive area of signal processing. Application of these methods to image and video compression has been very successful. However, their use for signal restoration problems (e.g. image denoising, deconvolution, etc.) has been extremely limited. A breakthrough in the use of these methods for noise removal and image deconvolution was recently reported by the authors [1]. We present these new solutions, showing their potential for several image estimation problems. The demonstrated results on many occasion overcome the best achievements in the field.

Bibliography

[1] A. Foi, K. Dabov, V. Katkovnik, and K. Egiazarian, *Shape-adaptive DCT for denoising and image reconstruction*, Electronic Imaging, Image Process.: Algorithms and Systems V **6064A-18** (January 2006).

[2] M. Gilge, T. Engelhardt, and R. Mehlan, *Coding of arbitrarily shaped image segments based on a generalized orthogonal transform*, Signal Processing: Image Communication **1** (October 1989), no. 2, 153–180.

[3] N. Saito, *Geometric harmonics as a statistical image processing tool for images defined on irregularly-shaped domains*, Proc. IEEE Statistical Signal Processing Workshop (Bordeaux, France), July 17–20, 2005.

[4] T. Sikora, *Low complexity shape-adaptive DCT for coding of arbitrarily shaped image segments*, Signal Process.: Image Comm. **7** (1995), 381–395.

*This work was partially supported by the Academy of Finland, project No. 213462 (Finnish Centre of Excellence program 2006 - 2011).

†alessandro.foi@tut.fi
‡vladimir.katkovnik@tut.fi
§karen@cs.tut.fi

Fast reconstruction algorithms for sparse multivariate and vector valued data. Applications in image processing and art restoration.*

M. Fornasier†
Johann Radon Institute for Computational and Applied Mathematics
Austrian Academy of Sciences, Linz (Austria)

On 11^{th} March 1944, a group of bombs launched from an Allied airplane hit the famous Italian Eremitani's Church in Padua, destroying it together with the priceless frescoes by A. Mantegna. Attempts were done to restore the fragments of these frescoes by traditional methods, without much success. A fast, robust, and efficient pattern recognition algorithm has been developed [4] in order to detect the right position and orientation of the fragments, by means of comparisons with an old gray level image of the fresco prior to the damage. Unfortunately what we can currently reconstruct is just a *fraction* of this priceless artwork. In particular, the *original color* of the missing parts is not known. In [2, 3] a novel method based on multivariate interpolation and variational calculus has been proposed for the recovery of the missing colors, from the data of the colors of detected fragments and the gray levels of the original pictures. Inspired by this problem, we present in this talk new developments of the algorithms introduced in [1] with an application to the fresco color restoration.

Bibliography

[1] I. Daubechies, M. Defrise, and C. De Mol, *An iterative thresholding algorithm for linear inverse problems with a sparsity constraint*, Commun. Pure Appl. Math. **57** (2004), no. 11, 1413–1457.

[2] M. Fornasier, *Nonlinear projection recovery in digital inpainting for color image restoration*, J. Math. Imaging Vis. (April17, 2006), DOI: 10.1007/s10851-006-4242-1, (online first), to appear.

[3] M. Fornasier and R. March, *Restoration of color images by vector valued BV functions and variational calculus*, preprint, 2006.

[4] M. Fornasier and D. Toniolo, *Fast, robust and efficient 2D pattern recognition for re-assembling fragmented digital images*, Pattern Recognition **38** (2005), no. 11, 2074–2087.

*The talk will present joint results with Riccardo March, Holger Rauhut, and Domenico Toniolo.
†massimo.fornasier@oeaw.ac.at

Structured matrix methods for computations with orthogonal rational functions

D. Fasino*
Department of Mathematics
and Computer Science
University of Udine (Italy)

L. Gemignani†
Department of Mathematics
University of Pisa (Italy)

Orthogonal polynomials on the real line satisfy a certain three-term recurrence relation and, therefore, they may be regarded as characteristic polynomials of an associated *tridiagonal matrix*. This enables the reduction of polynomial computations into a numerical linear algebra setting where very effective matrix methods can be applied. In particular, the exploitation of the tridiagonal structure yields a dramatic reduction of the computational costs, for example, in the computation of classical Gauss-type quadature rules.

In this talk we show that *orthogonal rational functions* can also benefit from a similar interplay between functional and matrix computations. In [2, 5] it was proved that rational functions with prescribed poles on the extended real line or on the unit circle that are orthogonal w.r.t. a discrete or a continuous scalar product can still be characterized by a three-term recurrence relation. Moreover, the numerator polynomials may also be regarded as the characteristic polynomials of a suitable structured matrix which has the form of a *diagonal-plus-semiseparable matrix* (dpss matrix for short).

A number of results obtained recently by the authors [1, 3, 4, 5] provide a set of tools for the efficient numerical treatment of dpss matrices. For example, the computation of Gauss-type quadrature rules for a prescribed set of rational functions can be performed by solving an eigenvalue problem for the associated dpss matrix. The application of these tools in the framework of the orthogonal rational function theory leads to numerically robust methods with reduced complexity.

Time permitting, at the end of the talk some open issues will be presented which can represent an active field for a joint research between the functional approximation and the numerical linear algebra community.

Bibliography

[1] D. Bini, L. Gemignani, and V.Y. Pan, *Fast and stable QR eigenvalue algorithms for generalized companion matrices and secular equations*, Numer. Math. **100** (2005), 373–408.

[2] A. Bultheel, P. González-Vera, E. Hendriksen, and O. Njåstad, *Orthogonal rational functions and tridiagonal matrices*, J. Comput. Appl. Math. **153** (2003), 89–97.

[3] D. Fasino, *Rational Krylov matrices and QR steps on Hermitian diagonal-plus-semiseparable matrices*, Numer. Linear Algebra Appl. **12** (2005), 743–754.

[4] D. Fasino and L. Gemignani, *Direct and inverse eigenvalue problems for diagonal-plus-semiseparable matrices*, Numer. Algorithms **34** (2003), 313–324.

[5] M. Van Barel, D. Fasino, L. Gemignani, and N. Mastronardi, *Orthogonal rational functions and structured matrices*, SIAM J. Matrix Anal. Appl. **26** (2005), 810–829.

*fasino@dimi.uniud.it
†gemignan@dm.unipi.it

Constructive Extremal Problems related to Inverse Balayage

M. Götz*
Continental Temic, Ingolstadt (Germany) and
K.U. Eichstätt-Ingolstadt (Germany)

Suppose G is a body in $\mathbb{R}^d$, $D \subset G$ is compact, and ρ a unit measure on ∂G. Inverse balayage refers to the question whether there exists a measure ν supported inside D such that ρ and ν produce the same electrostatic field outside G. Using linear optimization techniques to establish a duality principle between two extremal problems it is shown that such an inverse balayage exists if and only if

$$\sup_{\mu} \left\{ \inf_{y \in D} U^{\mu}(y) - \int U^{\rho} \, d\mu \right\} = 0 \,,$$

where the supremum is taken over all unit measures μ on ∂G and U^{μ} denotes the electrostatic potential of μ. A consequence is that pairs (ρ, D) admitting such an inverse balayage can be characterized by a ρ-mean-value principle, namely,

$$\sup_{z \in D} h(z) \geq \int h \, d\rho \geq \inf_{z \in D} h(z)$$

for all h harmonic in G and continuous up to the boundary.

Two approaches for the construction of an inverse balayage related to extremal point methods are presented, and the results are applied to problems concerning the determination of restricted Chebychev constants in the theory of weighted polynomial approximation.

The talk relates to work by [3, 4] and [2, 1].

Bibliography

[1] M. Götz, *Maximin Polynomials and Inverse Balayage.*

[2] ———, *On Extremal Problems related to Inverse Balayage.*

[3] B. Gustafsson, *On mother bodies of convex polyhedra*, SIAM J. Math. Anal. **29** (1998), 1106–1117.

[4] T. Sjödin, *Quadrature Identities and Deformation of Quadrature Domains*, Operator Theory: Advances and Applications, vol. 156, Birkhäuser Verlag, 2005.

*Mario.Goetz@temic.com

Reconstruction of a Polygon from its Moments

G. Golub*
Department of Computer Science
Stanford University, CA (USA)

Computation of certain kinds of numerical quadratures on polygonal regions of the plane and the reconstruction of these regions from their moments can be viewed as dual problems. In fact, this is a consequence of a little-known result of Motzkin and Schoenberg. In this talk, we discuss this result and address the inverse problem of (uniquely) reconstructing a polygonal region in the complex plane from a finite number of its complex moments. Algorithms have been developed for polygon reconstruction from moments and have been applied to tomographic image reconstruction problems. The numerical computations involved in the algorithm can be very ill-conditioned. We have managed to improve the algorithms used, and to recognize when the problem will be ill-conditioned. Some numerical results will be given.

Joint work with Peyman Milanfar and James Varah

*golub@sccm.stanford.edu

A triple-adaptive quadrature method based on the combination of the Ninomiya and the FLR schemes

T. Hasegawa[*], S. Hibino, Y. Hosoda
Department of Information Science
University of Fukui (Japan)

I. Ninomiya
Nagoya University (Japan)

An improvement of an adaptive Newton-Cotes quadrature method is proposed. Combining an adaptive Newton-Cotes scheme due to Ninomiya (1980) [3] and a doubly adaptive algorithm due to Favati, Lotti and Romani (1991) (abbreviated to FLR)[1] yields an efficient automatic quadrature method for univariate integration.

Ninomiya's method has a scheme to detect and treat analytically some singularities such as algebraic, discontinuous and logarithmic ones in the process of the successive bisection of the integration interval. On the other hand, the FLR method is effective particularly for oscillatory integrals because of the doubly adaptive algorithm based on the recursive monotone stable formulas [2].

Some numerical examples demonstrate the performance of the present quadrature method.

Bibliography

[1] P. Favati, G. Lotti, and F. Romani, *ALGORITHM 691 Improving QUADPACK automatic integration routines*, ACM Trans. Math. Software **17** (1991), 218–232.

[2] ———, *Interpolatory integration formulas for optimal composition*, ACM Trans. Math. Software **17** (1991), 207–217.

[3] I. Ninomiya, *Improvements of adaptive Newton-Cotes quadrature methods*, J. Information Processing **3** (1980), 162–170.

[*] hasegawa@fuis.fuis.fukui-u.ac.jp

Thin Plate Spline Interpolation on the Unit Interval

S. Hubbert*
School of Economics, Mathematics and Statistics
Birkbeck University of London (United Kingdom)

It is known that the thin plate spline interpolant to a sufficiently smooth function sampled at the scaled integers $h \cdot \mathbf{Z}$ converges at an optimal rate of h^3. However, when the function is sampled at equally spaced points on an interval then the known theoretical results predict a drop in the convergence rate from 3 to 3/2. In this talk we will present results from a recent numerical investigation of this situation. We will show, for instance, that there are functions for which the interpolant converges at a rate of 5/2. This motivates the question of how to characterize these functions which exhibit a faster convergence order than the theory currently predicts. Together with co-workers at the University of Göttingen, work has started on trying to answer this question analytically.

*s.hubbert@bbk.ac.uk

On determining the dimension of the bivariate spline space $S_n^1(\triangle)$*

G. Jaklič†
Institute of Mathematics, Physics and Mechanics
University of Ljubljana (Slovenia)
and Primorska Institute of Natural Sciences and Technology
University of Primorska, Koper (Slovenia)

J. Kozak ‡
Department of Mathematics
and Institute of Mathematics, Physics and Mechanics
University of Ljubljana (Slovenia)

In this talk the well-known problem of determining the dimension of the bivariate spline space of cubic C^1 splines $S_3^1(\triangle)$ will be revisited. By using the blossoming approach [1] and study of ranks of certain matrices [2] we show that the dimension $\dim S_3^1(\triangle)$ equals Schumaker's lower bound [3] for a large class of triangulations. An algorithm for determining whether a given triangulation belongs to such a class, will be presented, and the results generalized for splines of degree $n \geq 3$.

Bibliography

[1] Z.B. Chen, Y.Y. Feng, and J. Kozak, *The Blossom Approach to the Dimension of the Bivariate Spline Space*, J. Comput. Math. **18** (2000), 183–199.

[2] G. Jaklič, *On the dimension of the bivariate spline space $S_3^1(\triangle)$*, Int. J. Comput. Math. **82** (2005), 1355–1369.

[3] L.L. Schumaker, *Bounds on the dimension of spaces of multivariate piecewise polynomials*, Rocky Mountain J. of Math. **14** (1984), 251–265.

*Work supported by Ministry of Higher Education, Science and Technology of Slovenia.
†Gasper.Jaklic@fmf.uni-lj.si
‡Jernej.Kozak@fmf.uni-lj.si

On approximation of exponential sums in certain physical problems

E.A. Karatsuba*
Dorodnicyn Computing Center
Russian Academy of Sciences, Moscow (Russia)

The solutions of many problems from such fields of physics as quantum mechanics, quantum optics, solid state physics and wave processes are representable in the form of the exponential series with the general term $\varphi(n)\exp(2\pi i f(n))$, where $\varphi(x)$ and $f(x)$ are real functions of real argument, $i^2 = -1$. One of the methods of calculation of the solution of such a problems is to approximate the sum S,

$$S = \sum_{a<n\le b} \varphi(n)\exp(2\pi i f(n)),$$

where a and b take any values, including $+\infty$ and $-\infty$, by another sum having smaller number of summands. The Poisson summation formula permits to replace the sum S by the sum of some integrals. When $\varphi(x)$ and $f(x)$ satisfy certain conditions, these integrals are calculated with good accuracy, and as a result, the sum S can be replaced with good accuracy by another sum S_1,

$$S_1 = \sum_{\alpha<k\le\beta} \Phi(k)e^{2\pi i F(k)},$$

with the length $\beta - \alpha$, which is much smaller then $b - a$.

The problem of the approximation of the sums like S is investigated by many mathematicians, especially by number-theorists (see [3, 7, 8, 4]), since the exponential sums appear in such important problems of number theory as the analysis of the Riemann zeta function, the equations in integers (the additive problems) and the estimating of the number of integer points in the plane and many-dimensional domains. However, these results weren't widely known to the physicists. The papers (see [1, 2]), where the number-theoretic methods are applied to certain problems of physics, appeared only in the last years.

The purpose of my talk is to demonstrate the application of the theorem on the approximation of the exponential sums by shorter ones to the solution of the problem on the oscillations of a harmonic oscillator caused by quasiperiodic pushes. The results are published in [6, 5].

Bibliography

[1] E. Chassande-Mottin and A. Pai, *Best Chirplet chain: Near-optimal detection of gravitational wave chirps*, Physical review D **73** (2006), 0420031–04200323.

[2] M. Fleischhauer and W.P. Schleich, *Revivals made simple: Poisson summation formula as a key to the revivals in the Jaynes-Cummings model*, Physical review A (1993), no. 3, 4258–4269.

*ekar@ccas.ru

[3] G.H. Hardy and J.E. Littlewood, *Contributions to the theory of Riemann Zeta-Function and the theory of the distribution of primes*, Acta Math. **41** (1918), 119–196.

[4] A.A. Karatsuba, *Approximation of exponential sums by shorter ones*, Proc. Indian Acad. Sci. (Math. Sci.), vol. 97, 1987, pp. 167–178.

[5] E.A. Karatsuba, *Approximation of exponential sums in the problem on the oscillator motion caused by pushes*, in publl.

[6] ———, *Approximation of sums of oscillating summands in certain physical problems*, J. of Math. Phys. **45** (2004), 4310–4321.

[7] J.G. Van der Corput, *Verschärfung der abschätzung beim teilerproblem*, Math. Ann. **87** (1922), 39–65.

[8] I.M. Vinogradov, *On the average value of the number of classes of purely root form of the negative determinant*, Communications of Kharkhov Mathematics Society **16** (1917), 10–38.

Multidimensional local polynomial approximations with adaptive order and support*

V. Katkovnik†, A. Foi‡, J. Astola§
Institute of Signal Processing
Tampere University of Technology (Finland)

The pointwise local polynomial approximations (LPA) were intensively discussed in the last years. We refer to the book [1] for a nice and detailed overview of local polynomial modeling and related literature.

Recently the problem of pointwise adaptive estimation has received a powerful impetus in connection with a number of new methods developed for adaptive window-size selection. In this approach the estimates are calculated and compared with the main intention to select the best window-size for each point. Various developments of this idea and various statistical rules are a subject of a thorough study in the papers [6, 3]. These sort of methods can be treated as a quality-of-fit statistics applied locally in pointwise manner and known as Lepski's approach. Multiple results have been reported on successful development of this sort of technique for image processing [4, 5, 2].

It is important to note that in the Lepski approach the order of the LPA is fixed. In this talk, we discuss different ideas and approaches leading to the LPA with varying and data-adaptive orders. Another important element of this novel approach is that the LPA becomes non-local involving adaptively selected parts of the data demonstrating some similarities in their properties.

Our approach to estimation for the point x_0 can be roughly described as the following three stage procedure.

Stage I: For every $\tilde{x} \in X$, define an adaptive neighborhood $G_{\tilde{x}}$ of $\tilde{x}$ where a simple (e.g. constant or linear) signal model fits to data;

Stage II: Apply an higher-order orthogonal polynomial approximation to the data in the neighborhood and use a model selection procedure in order to identify non-zero elements (thus the order) of the polynomial model. Calculate the corresponding estimates $y_{G_{\tilde{x}}}(x)$ of the function for all $x \in G_{\tilde{x}}$. These $y_{G_{\tilde{x}}}$ are calculated for all $\tilde{x} \in X$.

Stage III: Let I_{x_0} be a set of $G_{\tilde{x}}$ having a common point at $x = x_0$, then the estimate $\hat{y}(x_0)$ is calculated as an aggregate of $y_{G_{\tilde{x}}}(x_0)$, $G_{\tilde{x}} \in I_{x_0}$. This aggregation can be calculated as the mean or the weighted mean of the estimates $y_{G_{\tilde{x}}}(x_0)$, $G_{\tilde{x}} \in I_{x_0}$, with the weights depending on the size of the neighborhoods $G_{\tilde{x}} \in I_{x_0}$ and the quality (variance) of the transform estimate $y_{G_{\tilde{x}}}(x_0)$.

We present a number of novel and well tested algorithms implementing the above ideas of the novel approach to nonparametric regression estimation. The proposed algorithms are quite general and can be applied for the solution of various scientific and technical problems.

Bibliography

[1] J. Fan and I. Gijbels, *Local polynomial modelling and its applications*, Chapman & Hall, London, 1996.

[2] A. Foi, *Anisotropic nonparametric image processing: theory, algorithms and applications*, Ph.D. thesis, Dipartimento di Matematica, Politecnico di Milano, Italy, 2005.

---

*This work was partially supported by the Academy of Finland, project No. 213462 (Finnish Centre of Excellence program 2006 - 2011).

†vladimir.katkovnik@tut.fi
‡alessandro.foi@tut.fi
§jaakko.astola@tut.fi

[3] A. Goldenshluger and A. Nemirovski, *On spatial adaptive estimation of nonparametric regression*, Math. Meth. Statistics **6** (1997), 135–170.

[4] V. Katkovnik, K. Egiazarian, and J. Astola, *Adaptive window size image de-noising based on intersection of confidence intervals (ICI) rule*, Journal of Mathematical Imaging and Vision **16** (2002), 223–235.

[5] V. Katkovnik, A. Foi, K. Egiazarian, and J. Astola, *Directional varying scale approximations for anisotropic signal processing*, Proceedings of EUSIPCO 2004, 2004, pp. 101–104.

[6] O. Lepski, E. Mammen, and V. Spokoiny, *Ideal spatial adaptation to inhomogeneous smoothness: an approach based on kernel estimates with variable bandwidth selection*, The Annals of Statistics **25** (1997), no. 3, 929–947.

Fast evaluation of quadrature formulae on the sphere

J. Keiner*
Institute of Mathematics
University of Lübeck (Germany)

D. Potts†
Faculty of Mathematics
Chemnitz University of Technology (Germany)

Recently, a fast approximate algorithm for the evaluation of expansions in terms of standard $\mathrm{L}^2\left(\mathbb{S}^2\right)$-orthonormal spherical harmonics at arbitrary nodes on the sphere $\mathbb{S}^2$ has been proposed in [3]. Our aim is to develop a fast algorithm for the adjoint problem, hence the computation of expansion coefficients from sampled data by means of quadrature rules.

We give a formulation in matrix-vector notation and an explicit factorisation of the corresponding spherical Fourier matrix that is based on the first algorithm. Starting from this factorisation, we obtain the corresponding adjoint factorisation and are able to implement the corresponding transform. This 'adjoint' algorithm can be employed to evaluate quadrature rules for arbitrary quadrature nodes and weights on the sphere $\mathbb{S}^2$.

We provide results of test computations with respect to stability, accuracy and performance of the obtained algorithm. As examples, we consider a variety of proposed test functions using classical Gauß-Legendre and Clenshaw-Curtis quadrature rules. Furthermore, we also consider an equidistribution from [1] and the HEALPix pixelation scheme ([2]), each with equal weights for all nodes to obtain a convenient quadrature rule. Especially the HEALPix scheme has great relevance as data storage standard in certain applications like cosmic microwave background estimation.

Bibliography

[1] W. Freeden, T. Gervens, and M. Schreiner, *Constructive Approximation on the Sphere*, (1998).

[2] K.M. Górski, E. Hivon, A.J. Banday, B.D. Wandelt, F.K. Hansen, M. Reinecke, and M. Bartelmann, *Healpix: A framework for high-resolution discretization and fast analysis of data distributed on the sphere*, The Astrophysical Journal **622** (2005), 759?–771.

[3] S. Kunis and D. Potts, *Fast spherical fourier algorithms*, J. Comput. Appl. Math. **161** (2003), 75–98.

*keiner@math.uni-luebeck.de
†potts@mathematik.tu-chemnitz.de

On density of homogeneous polynomials on star-like and convex surfaces

A. Kroó*
Alfréd Rényi Institute of Mathematics
Hungarian Academy of Sciences, Budapest (Hungary)

In this talk we shall discuss the density of homogeneous polynomials in the space of continuous functions on star-like and convex surfaces. Weierstrass-type density results and Jackson-type estimates will be presented.

*kroo@renyi.hu

Interpolation of scattered data on the sphere by localised polynomials

S. Kunis*
Faculty of Mathematics
Chemnitz University of Technology (Germany)

Let $\mathbb{S}^2 := \{x \in \mathbb{R}^3 : \|x\|_2 = 1\}$ and Y_k^n denote the unit sphere and the spherical harmonics, respectively. For given samples $(\xi_j, y_j) \in \mathbb{S}^2 \times \mathbb{C}$ and chosen weights $\hat{w}_k > 0$, we are interested in solving the constrained optimisation problem

$$\sum_{k=0}^{N} \sum_{n=-k}^{k} \frac{|\hat{f}_k^n|^2}{\hat{w}_k} \to \min \quad \text{subject to} \quad \sum_{k,n} \hat{f}_k^n Y_k^n(\xi_j) = y_j,$$

$j = 0, \ldots, M-1$, for the unknown Fourier coefficients $\hat{f}_k^n$. This problem is shown to be equivalent to solving

$$\sum_{l=0}^{M-1} \tilde{f}_l K_N(\xi_j, \xi_l) = y_j, \qquad j = 0, \ldots, M-1,$$

for the coefficients $\tilde{f}_l$, where $K_N(\xi_j, \xi_l) = \sum_{k=0}^{N} \hat{w}_k P_k(\xi_j^\top \xi_l)$ denotes the zonal polynomial to the weights $\hat{w}_k$.

We present an explicit construction of smooth weights $\hat{w}_k$, such that the polynomial K_N is well localised. Thus, the interpolation matrix $K_N(\xi_j, \xi_l)$ is diagonal dominant for every polynomial degree $N > Cq^{-1}$, if the sampling nodes ξ_j are separated by $q := \min_{j \neq l} \arccos(\xi_j^\top \xi_l)$. In particular, the proposed iterative scheme using the nonequispaced FFT on the sphere takes only $\mathcal{O}(M \log^2 M)$ arithmetic operations for the interpolation at M quasi-uniform nodes.

*kunis@mathematik.tu-chemnitz.de

Generation of Radau-Kronrod and Lobatto-Kronrod quadrature formulas

D. Laurie*
Department of Mathematical Sciences
University of Stellenbosch (South Africa)

A method for computing Kronrod extensions of n-point Radau and Lobatto rules in $O(n^2)$ operations is presented. The method is applicable to any weight function for which enough three-term recursion coefficients are known.

*dpl@sun.ac.za

Monotonicity of Jacobi polynomials and positive quadrature on the sphere*

P.C. Leopardi†
School of Mathematics
University of New South Wales, Sydney (Australia)

W. Gautschi ‡
Department of Computer Sciences
Purdue University, Indiana (USA)

Reimer [4, 5] proved that a positive weight quadrature rule on the unit sphere $\mathbb{S}^d \subset \mathbb{R}^{d+1}$ has the property of quadrature regularity (Sloan and Womersley, [6]). Hesse and Sloan [2, 3] used a related property, called Property (R) in their work on estimates of quadrature error on $\mathbb{S}^d$.

This talk will cover the following recent related conjectures and results:

1. For the normalized the Jacobi polynomial $\widetilde{P}_t^{(\alpha,\beta)}(x) := P_t^{(\alpha,\beta)}(x) \ / \ P_t^{(\alpha,\beta)}(1)$ it is conjectured that if $0 < \widetilde{P}_1^{(\alpha,\beta)}(\cos\theta) < \widetilde{P}_2^{(\alpha,\beta)}(\cos\theta/2)$ for $\theta \in (0, \Theta_1^{(\alpha,\beta)})$ then for $\theta \in (0, \Theta_t^{(\alpha,\beta)})$, for all $t \geqslant 1$,

$$0 < \widetilde{P}_t^{(\alpha,\beta)}\left(\cos\frac{\theta}{t}\right) < \widetilde{P}_{t+1}^{(\alpha,\beta)}\left(\cos\frac{\theta}{t+1}\right).$$

   where $\cos\Theta_t^{(\alpha,\beta)}$ is the largest zero of $P_t^{(\alpha,\beta)}$. This conjecture is joint work with Walter Gautschi.

2. The Sturm comparison theorem and some of the estimates of Gatteschi [1] can be used to prove a number of results on Jacobi polynomials which are weaker than the conjecture, but strong enough to enable the constants related to Property (R) to be estimated.

3. A variation, using Jacobi polynomials of the form $P_t^{(1+d/2,d/2)}$, on Reimer's [5, Lemma 6.23] bound on the sum of the quadrature weight within a spherical cap, allows the constants for Property (R) to be estimated.

Bibliography

[1] L. Gatteschi, *New inequalities for the zeros of Jacobi polynomials*, SIAM J. Math. Anal. **18** (1987), 1549–1562.

[2] K. Hesse and I.H. Sloan, *Worst-case errors in a Sobolev space setting for cubature over the sphere S^2*, Applied Reports AMR03/19, The University of New South Wales, 2003.

*Work supported by the University of New South Wales and the Australian Research Council Centre of Excellence for Mathematics and Statistics of Complex Systems (MASCOS)

†paul.leopardi@unsw.edu.au
‡wxg@cs.purdue.edu

[3] ———, *Cubature over the sphere S^2 in Sobolev spaces of arbitrary order*, Applied Reports AMR04/27, The University of New South Wales, 2004.

[4] M. Reimer, *Hyperinterpolation on the sphere at the minimal projection order*, J. Approx. Theory **104** (2000), 272–286.

[5] ———, *Multivariate polynomial approximation*, International Series of Numerical Mathematics, vol. 144, Birkhäuser Verlag, Basel, 2003.

[6] I.H. Sloan and R.S. Womersley, *Constructive polynomial approximation on the sphere*, J. Approx. Theory **103** (2000), 91–118.

Polynomial Sampling of Fractal Measures: I.F.S.–Padé Approximants

G. Mantica*
Center for Nonlinear and Complex Systems
University of Insubria, Como (Italy)

Constructing the Jacobi matrix of a system of orthogonal polynomials on the unit interval leads naturally to a discrete approximation of the orthogonality measure by a sum of atomic measures. Despite the fact that the numerical computation of the Jacobi matrix may require considerable effort [4] and despite the marvelous convergence properties of Gaussian integration, this approximation is rather crude, especially when the sampled measure is singular continuous [6] and one is looking for estimates of its multi-fractal properties.

This limitation can be overcome by finding a system of iterated functions with probabilities (I.F.S.) [1] and the associated invariant measure whose Jacobi matrix coincides, up to finite order, with that of the sampled measure.

I adopt a variant of this classical problem—working with affine maps with equal contraction ratios [3, 5, 2] for which I present here, for the first time to the best of my knowledge, a stable solution algorithm. I show numerical examples with emphasis on the study of the singularity structure of the sampled measure and of the asymptotics of its Fourier transform (Fourier–Bessel functions) [7].

Bibliography

[1] M.F. Barnsley and S.G. Demko, *Iterated function systems and the global construction of fractals*, Proc. R. Soc. London, vol. A 399, 1985, pp. 243–275.

[2] D. Bessis and S. Demko, *Stable recovery of fractal measures by polynomial sampling*, Physica D **47** (1991), 427–438.

[3] J. Elton and Z. Yan, *Approximation of measures by Markov processes and homogeneous affine iterated function systems*, Constr. Approx. **5** (1989), 151–170.

[4] W. Gautschi, *Orthogonal Polynomials, Computation and Approximation*, Oxford Univ. Press, 2004.

[5] C.R. Handy and G. Mantica, *Inverse Problems in Fractal Construction: Moment Method Solution*, Physica D **43** (1990), 17–36.

[6] G. Mantica, *A Stieltjes Technique for Computing Jacobi Matrices Associated With Singular Measures*, Constr. Appr. **12** (1996), 509–530.

[7] G. Mantica and D. Guzzetti, *The Asymptotic Behaviour of the Fourier Transforms of Orthogonal Polynomials II: L.I.F.S. measures and Quantum Mechanics*, Annales Henri Poincaré (2006), preprint mp–arc 04–361, accepted for publication.

*giorgio@uninsubria.it

Meshless methods and numerical integration rules with applications to axisymmetric geomechanical problems*

M. Ferronato†, **A. Mazzia**‡, G. Pini§, G. Gambolati¶
Department of Mathematical Methods and Models in Applied Sciences
University of Padua (Italy)

Meshless methods, such as the Meshless Local Petrov-Galerkin (MLPG) method[1, 2] are used with the goal of achieving computational efficiency in a mesh-free procedure. They recently received an increasing attention due to their flexibility in solving several engineering problems, especially with reference to discontinuities, moving boundaries, or complex non-standard geometries.

Numerical integration plays an important role for the accuracy and convergence of the meshless numerical solution. In this communication we report several numerical cubature rules in order to both reduce significantly the computational cost and increase the solution accuracy. In particular, formulas based on Gauss-Legendre rule with mapping on polar coordinates, cubature rules and midpoint quadrature formulas are analyzed[5, 3] and compared.

The use of MLPG with the most efficient integration rules is considered for the solution of structural problems. Solution of elastic structural problems has been already investigated by comparing the computational behavior and numerical accuracy of MLPG and Finite Element technique [4]. The present communication addresses the performance and accuracy of MLPG in the solution of axisymmetric geomechanical problems.

Bibliography

[1] S.N. Atluri and S. Shen, *The Meshless Local Petrov Galerkin (MLPG) Method*, Tech Science Press, Forsyth, GA, USA, 2002.

[2] ______, *The Meshless Method (MLPG) for Domain and BIE Discretizations*, Tech Science Press, Forsyth, GA,USA, 2004.

[3] S. De and K.J. Bathe, *The method of finite spheres with improved numerical integration*, Comp. & Struct. **79** (2001), 2183–2196.

[4] M. Ferronato, A. Mazzia, G. Pini, and G. Gambolati, *Meshless Local Petrov-Galerkin Method and standard unstructured Finite Elements for the solution of 2-D elastic structural problems.*

[5] W.H. Peirce, *Numerical integration over the planar annulus*, J. Soc. Indust. Appl. Math. **5** (1957), 66–73.

*Work supported by Italian MIUR project "Numerical Models for Multiphase Flow and Deformation in Porous Media".

†ferronat@dmsa.unipd.it
‡mazzia@dmsa.unipd.it
§pini@dmsa.unipd.it
¶gambo@dmsa.unipd.it

Optimally Localizing Approximate Identities on the 2–sphere — an Alternative Approach

V. Michel*
Department of Mathematics
University of Kaiserslautern (Germany)

The talk concerns optimally localizing approximate identities on the 2–sphere. Several authors (Laín Fernandez, Prestin, Simons et al.) have discussed measures of localization on the sphere and functions that minimize those. We will introduce an alternative approach that measures the localization of a spherical radial basis function via a weight function w that can control the strength of localization.
Numerical tests show that the obtained kernels are not establishing an approximate identity, i.e. we do not observe a converge of the Legendre coefficients to the constant (1)–sequence. For this reason we combine the localization measure with another functional that computes the deviation from an approximate identity. This combined functional then yields approximate–identity–families of strongly localizing kernels on $[-1,1]$ ($S^2 \times S^2$, respectively). Existence and uniqueness results as well as a convergence theorem can be proved. Moreover, we distinguish bandlimited and non–bandlimited kernels.

*michel@mathematik.uni-kl.de

Error Bounds for Gauss Type Quadrature Formulae of Analytic Function

S.E. Notaris*
Department of Mathematics
University of Athens (Greece)

We compare two methods, which are widely used in order to obtain error bounds for Gauss type quadrature formulae (including Gauss-Radau and Gauss-Lobatto formulae) of analytic functions, proposed the first in G. Hämmerlin, "Fehlerabschätzung bei numerischer Integration nach Gauss" in (B. Brosowski and E. Martensen, eds.), Methoden und Verfahren der mathematischen Physik, vol. 6, Bibliographisches Institut, Mannheim-Wien-Zürich, 1972, pp. 153-163, and the second in W. Gautschi and R.S. Varga, "Error bounds for Gaussian quadrature of analytic functions", *SIAM J. Numer. Anal.*, v. 20, 1983, pp. 1170-1186.

*notaris@math.uoa.gr

On differential properties for multivariate orthogonal polynomials*

M. Piñar[†], M.A. de Morales[‡], L. Fernández[§]
Department of Applied Mathematics
University of Granada (Spain)

T. Pérez[¶]
Institute "Carlos I" for Theoretical
and Computational Physics
University of Granada (Spain)

In this paper, orthogonal polynomials in several variables are studied. We consider multivariate orthogonal polynomials whose gradients satisfy some quasi-orthogonality conditions. Several characterizations for these polynomials are obtained. These characterizations include the analogous of the semiclassical Pearson differential equation, the structure relation and a partial differential equation, whose coefficients depend on the total degree of the polynomials. Some examples are given.

Bibliography

[1] L. Fernández, T.E. Pérez, and M.A. Piñar, *Weak Classical Orthogonal Polynomials in two variables*, J. Comput. Appl. Math. **178** (2005), 191–203.

[2] P.K. Suetin, *Orthogonal Polynomials in Two Variables*, Gordon and Breach, Amsterdam, 1999.

[3] Y. Xu, *On multivariate orthogonal polynomials*, SIAM J. Math. Anal. **24** (1993), 783–794.

*Work supported by Ministerio de Ciencia y Tecnología (MCYT) of Spain and by the European Regional Development Fund (ERDF) through the grant MTM2005-08646-C02-02

[†]mpinar@ugr.es
[‡]alvarezd@ugr.es
[§]lidiafr@ugr.es
[¶]tperez@ugr.es

Exponentially localized polynomial frames on the unit interval and the Euclidean sphere

J. Prestin[*]
Institute of Mathematics
University of Lübeck (Germany)

H.N. Mhaskar[†]
Department of Mathematics
California State University
Los Angeles (USA)

In this work we present some unifying ideas of exponential localization for algebraic and spherical polynomials.

In more detail we describe an exponentially localized polynomial frame on the interval $[-1, 1]$, and on the unit sphere of a Euclidean space. Even though the frame coefficients may be computed using the coefficients of a function in an orthogonal polynomial expansion, the behavior of these coefficients near a point on the interval characterizes the possibility of an analytic continuation of the function in a complex neighborhood of the point in question.

Even though our main interest is in the characterization of local smoothness of a function f in terms of the sequence of Fourier coefficients $\{\hat{f}(k)\}$, from the point of view of computations, we will also describe our results when samples of the functions in question are available, instead of the coefficients.

Our construction allows one to construct exponentially localized kernels based only on some summability estimates. In turn, the localization enables us to obtain a characterization of local Besov spaces on the interval also in the case of some more general systems of orthogonal polynomials.

Bibliography

[1] H.N. Mhaskar and J. Prestin, *On local smoothness classes of periodic functions*, Journal of Fourier Analysis and Applications **11** (2005), no. 3, 353–373.

[2] ———, *Polynomial frames: a fast tour*, Approximation Theory XI: Gatlinburg 2004 (Brentwood) (C. K. Chui, M. Neamtu, and L. Schumaker, eds.), Nashboro Press, 2005, pp. 287–318.

[*] prestin@math.uni-luebeck.de
[†] hmhaska@calstatela.edu

The structure relations and difference representations for orthogonal polynomials of hypergeometric type in two discrete variables*

J. Rodal†, I. Area‡, E. Godoy§
Department of Applied Mathematics II
University of Vigo (Spain)

In [1], we provided a method for constructing orthogonal polynomials of two discrete variables on a simplex, giving a system of two independent recurrence relations in vector–matrix form, their second order partial difference equation, structure relations and some limit relations between some bivariate families (Hahn, Meixner, Kravchuk and Charlier) obtained using the method.

Motivated by the recent paper [2] where a systematic study of the orthogonal polynomial solutions of a second order partial difference equation of hypergeometric type of two variables was done and from the classification of the admissible equations of hypergeometric type, we have obtained an explicit difference derivative representation and structure relations for orthogonal polynomials of hypergeometric type in two discrete variables. This is explicitly discussed for bivariate Hahn, Meixner, Kravchuk and Charlier polynomials.

Bibliography

[1] J. Rodal, I. Area, and E. Godoy, *Orthogonal polynomials of two discrete variables on the simplex*, Integ. Transforms Special Func. **16** (2005), no. 3, 263–280.

[2] ———, *Linear partial difference equations of hypergeometric type: orthogonal polynomial solutions in two discrete variables*, J. Comput. Appl. Math. (2006), in press.

*Work supported by Ministerio de Ciencia y Tecnología of Spain, cofinanced by the European Community fund FEDER.

†jrodal@edu.xunta.es
‡area@dma.uvigo.es
§egodoy@dma.uvigo.es

The discrete pulse transform

C.H. Rohwer*
Department of Mathematical Sciences
University of Stellenbosch (South Africa)

The Haar Wavelet can be interpreted as decomposing a piecewise constant function into block pulses that come in pairs. In the mathematics of vision, as envisaged by Marr, this is not quite appropriate and a fast physiologically realisable decomposition into a more flexible class of pulses has been persued. The discrete pulse transform meets the requirements in one dimension. .

Bibliography

[1] D.P. Laurie and C.H. Rohwer, *The Discrete Pulse Transform*, SIAM J. Math. Anal., Accepted.

[2] C.H. Rohwer, *Nonlinear Smoothing and Multiresolution Analysis*, ISNM, vol. 150, Birkhäuser, 2005.

*chr@sun.ac.za

Spectral behavior of compact and Cesaro non-Hermitian perturbations of Hermitian (structured) sequences*

S. Serra-Capizzano†
Department of Physics and Mathematics
University of Insubria, Como (Italy)

Under the mild trace-norm assumptions, we show that the eigenvalues of a generic (non Hermitian) complex perturbation of a Hermitian matrix sequence distributed as $f(t)$ over a compact domain D have still the same distribution function. The result has application to the study of the asymptotic spectrum of Finite Differences, Finite Elements approximations of PDEs, to the related preconditioning problems, to the analysis of perturbations of Jacobi matrices etc. With regard to the latter problem, we have $f(t) = 2\cos t$ and $D = [0, \pi]$. As a consequence we prove that the real interval $[-2, 2]$ =Range$[2\cos(\cdot), D]$ is still a cluster for the asymptotic joint spectrum and, moreover, $[-2, 2]$ still attracts strongly (with infinite order) the perturbed matrix sequence. The results follow in a straightforward way from more general facts that we prove in an asymptotic linear algebra framework and are plainly generalized to the case of matrix-valued symbols, which arises when dealing with orthogonal polynomials with asymptotically periodic recurrence coefficients.

Bibliography

[1] L. Golinskii and S. Serra-Capizzano, *The asymptotic spectrum of non symmetrically perturbed symmetric jacobi matrix sequences*, J. Approx. Theory (2005), `http://arxiv.org/abs/math.SP/0512222`, submitted.

[2] R. Killip and B. Simon, *Sum rules for Jacobi matrices and their applications to spectral theory*, Ann. Math. **158** (2003), 253–321.

[3] S. Serra-Capizzano, *The GLT class as a Generalized Fourier Analysis and applications*, Linear Algebra Appl., to appear.

[4] ———, *Spectral behavior of matrix sequences and discretized boundary value problems*, Linear Algebra Appl. **337** (2001), 37–78.

[5] ———, *Generalized Locally Toeplitz sequences: spectral analysis and applications to discretized Partial Differential Equations*, Linear Algebra Appl. **366** (2003), no. 1, 371–402.

*Work supported by Swedish Science Council VR 2002-5532 and by the Italian MIUR, grant number 2003011441 and grant number 2004015437
†`stefano.serrac@uninsubria.it`

A Physical View on Quaternion Wavelets

L. Traversoni*
IPH Department
University Autónoma Metropolitana, Iztapalapa (Mexico)

From the many attempts made before to interpolate quaternions or to use them in the way of quaternion wavelets in none of them the true distinguishing attributes of quaternions, that is, their capacity to represent movements has been used, this paper is an attempt to do it. Historically quaternion wavelets were invented by me Traversoni [3] and there is also some better known paper by Mitrea [2] about Clifford Wavelets but like me at the beginning all the following authors like Bayro[1] only used quaternion wavelets to deal with images our proposal is to use them to represent movements.

Bibliography

[1] E. Bayro, *Image analysis using the phase concept of the quaternion wavelet transform*, Numerical Algorithms (2004).

[2] M. Mitrea, *Clifford Wavelets, Singular Integrals and Hardy Spaces*, Lecture Notes in Mathematics, vol. 1575, Springer Verlag, 1994.

[3] L. Traversoni, *Quaternion Wavelet Problems*, Proceedings of the VIII Texas International Symposium on Approximation Theory, 1994.

*ltd@xanum.uam.mx

Is Gauss quadrature better than Clenshaw–Curtis?

L.N. Trefethen*
Oxford University Computing Laboratory
University of Oxford (England)

We consider the question of whether Gauss quadrature, which is very famous, is more powerful than the much simpler Clenshaw–Curtis quadrature, which is less well-known. Seven-line MATLAB codes are presented that implement both methods, and experiments show that the supposed factor-of-2 advantage of Gauss quadrature is rarely realized. Theorems are given to explain this effect. First, following Elliott and O'Hara and Smith in the 1960s, the phenomenon is explained as a consequence of aliasing of coefficients in Chebyshev expansions. Then another explanation is offered based on the interpretation of a quadrature formula as a rational approximation of $\log((z+1)/(z-1))$ in the complex plane. Gauss quadrature corresponds to Padé approximation at $z=\infty$. Clenshaw–Curtis quadrature corresponds to an approximation whose order of accuracy at $z=\infty$ is only half as high, but which is nevertheless equally accurate near $[-1,1]$ for reasons related to the potential theoretic phenomenon of "balayage".

*LNT@comlab.ox.ac.uk

New spline basis functions for sampling approximations

T. Ueno[*], M. Okada[†]
Department of Mathematics
Tokyo Metropolitan University (Japan)

We propose a method of constructing new spline basis functions φ with compact support on $\mathbb{R}$. These spline basis functions consisting of a linear combination of the cardinal B-spline enable us to achieve simultaneously a good sampling approximation and an exact interpolation for a sufficiently smooth function f. For $f \in W^{N,p}(\mathbb{R})$, let $S_j(f,\varphi)$ be a interpolation of f based on φ defined as follows:

$$S_j(f,\varphi)(x) := \sum_{k\in\mathbb{Z}} f(2^{-j}k)\varphi(2^j x - k).$$

Then we have the following error estimate:

$$\|S_j(f,\varphi) - f\|_{L^p} \leq 2^{-jN} C_{\varphi,p,N} \|f^{(N)}\|_{L^p}, \qquad j = 1, 2, \ldots .$$

Furthermore, we establish new spline basis functions on an interval as well as on $\mathbb{R}$ which are adapted to the sampling approximation by using Newton's interpolation formula. We shall also discuss numerical applications based on φ.

Bibliography

[1] C.K. Chui and E. Quak, *Wavelets on a bounded interval*, Numerical methods in approximation theory **9** (1992), 53–75.

[2] C. de Boor, *A practical guide to splines*, Springer–Verlag, 1978.

[3] R. DeVore and G.G. Lorentz, *Constructive Approximation*, Springer–Verlag, 1993.

[4] H.L. Resnikoff and R.O. Jr. Wells, *Wavelet Analysis*, Springer–Verlag, 1998.

[5] T. Ueno, T. Ide, and M. Okada, *A wavelet collocation method for evolution equations with energy conservation property*, Bull. Sci. Math **127** (2003), 569–583.

[6] T. Ueno and M. Okada, *New spline basis functions for sampling approximations*, submitted for publication.

[*] tosshy@comp.metro-u.ac.jp
[†] moka@comp.metro-u.ac.jp

Exact rational minimax approximation and interpolation with prescribed poles

J. Van Deun*
Department of Computer Science
K.U. Leuven (Belgium)

We give explicit formulas for the rational functions with prescribed poles that best approximate the zero function according to the maximum norm on the interval $[-1, 1]$. These rational functions are generalisations of the well-known Chebyshev polynomials and share many of their properties, such as the equi-oscillatory behaviour and orthogonality relations. The zeros of these functions are ideally suited for rational interpolation. Furthermore, we give an electrostatic interpretation similar to the one for the zeros of Chebyshev polynomials.

*joris.vandeun@cs.kuleuven.be

Structured linear systems in shape reconstruction from moments

B. Verdonk*
Department of Mathematics and Computer Science
University of Antwerp (Belgium)

The problem of reconstructing a function and/or its domain given its moments is encountered in many areas. Several applications from diverse areas such as probability and statistics, signal processing, computed tomography and inverse potential theory (magnetic and gravitational anomaly detection) can be cited, to name just a few. Recently, the reconstruction of a multidimenstional shape from its moments has been solved in its full generality. As we shall explain, the reconstruction technique is based on the solution of structured linear systems. The applicability of structured solvers is being explored, with the aim of improving the efficiency and the robustness of the reconstruction algorithm.

Joint work with: Annie Cuyt, Mieke Schuermans, Sabine Van Huffel.

*brigitte.verdonk@ua.ac.be

Multivariate Jacobi polynomials with singular weights and the Bernstein operator

S. Waldron*
Department of Mathematics
University of Auckland (New Zealand)

I will discuss the basic structural properties of a new representation of Jacobi polynomials on a simplex. As an illustration, I will use it to show that the orthogonal projection of a continuous function on the Jacobi polynomials of a fixed degree has a limit as the parameters of the weight approach -1, and give an explicit formula for the corresponding 'orthogonal expansion'. It turns out that this expansion is closely related to the limiting eigenfunctions of the Bernstein operator.

*waldron@math.auckland.ac.nz

Explicit Error Formulas for Interpolatory Quadrature Rules for Rational Integrands*

J.A.C. Weideman†
Department of Mathematical Sciences
University of Stellenbosch (South Africa)

We derive explicit formulas for the error when rational functions such as $1/(x-c)$ and $1/(x^2+c^2)$ are integrated with interpolatory quadrature rules based on Chebyshev points (such as the Fejér or Clenshaw-Curtis formulas). These error formulas are then used to explain interesting convergence behaviour when the pole of the integrand is close to the interval of integration. Key to our analysis is an explicit formula we found for $\int_{-1}^{1} \frac{T_N(x)}{x-c}\, dx$, where $T_N(x)$ is the Chebyshev polynomial of degree N and c a constant not in $[-1,1]$. This formula, which can be expressed either in terms of Lerch's Φ function or the ${}_2F_1$ hypergeometric function, is not well known: It is not listed in any of the tables of integrals we have consulted and none of the symbolic software packages we tried was able to reproduce it.

*Work supported by NRF grant FA2005032300018.
†weideman@dip.sun.ac.za

Posters

On a generalized Lindelöf orthogonal polynomials with applications

E. Al-Aidarous*
Mathematics Department
King Abdul Aziz University, Jeddah (Saudi Arabia)

The numerical inversion of a specified complex integral transform which arises in the theory of linear viscoelasticity is known to be ill-posed. In this paper, we present a generalized Lindel öf orthogonal polynomials and give their main algebraic and computational properties. In particular, an explicit formula for the three recurrence relations is derived. We show then how such polynomials are used to construct approximations of the solution of the desired ill-posed inverse problem. Numerical experiments are given.

Keywords: Linear viscoelasticity, relaxation spectrum, ill-posedness, orthogonal polynomials.

*ealaidarous@kau.edu.sa

Bivariate Lagrange interpolation at the Padua points: computational aspects*

M. Caliari†, M. Vianello‡
Department of Pure
and Applied Mathematics
University of Padua (Italy)

S. De Marchi§, R. Montagna
Department of Computer Science
University of Verona (Italy)

In [1] we gave a simple, geometric and explicit construction of bivariate polynomial interpolation at certain points in the square (experimentally introduced and termed "Padua points" in [3]), and we showed that the associated Lebesgue constant has minimal order of growth $\mathcal{O}((\log n)^2)$; see also [2] for an algebraic setting and solution of the interpolation problem. Here we present an accurate and efficient implementation of the interpolation formula (based on the reproducing-kernel algorithm in [4]), accompanied by several numerical tests. We also extend the method to polynomial-based interpolation on bivariate compact domains with various geometries, via composition with suitable smooth transformations, and we present an application to surface compression.

Bibliography

[1] L. Bos, M. Caliari, S. De Marchi, M. Vianello, and Y. Xu, *Bivariate Lagrange interpolation at the Padua points: the generating curve approach*, J. Approx. Theory (2006), in press.

[2] L. Bos, S. De Marchi, M. Vianello, and Y. Xu, *Bivariate Lagrange interpolation at the Padua points: the ideal theory approach*, submitted, 2006.

[3] M. Caliari, S. De Marchi, and M. Vianello, *Bivariate polynomial interpolation on the square at new nodal sets*, Appl. Math. Comput. **165** (2005), no. 2, 261–274.

[4] M. Caliari, M. Vianello, S. De Marchi, and R. Montagna, *Hyper2d: a numerical code for hyperinterpolation on rectangles*, submitted, 2006.

*Work supported by the "ex-60%" funds of the Universities of Padua and Verona, and by the INdAM-GNCS.

†mcaliari@math.unipd.it
‡marcov@math.unipd.it
§demarchi@sci.univr.it

Approximate solution of singular integro-differential equations in Generalized Hölder spaces*

I. Caraus†
Faculty of Mathematics and Informatics
Moldova State University, Chisinau (Moldova)

Let Γ be an arbitrary smooth closed contour bounding a simply- connected region F^+ of complex plane containing the point $t = 0$. By F^- we denote the complement of $F^+ \cup \Gamma$. Let $z = \psi(w)$ be the Riemann function mapping conformably and unambiguously the outside of unit circle $\{|w| = 1\}$ on the F^-, so that $\psi(\infty) = \infty$, $\psi^{'}(\infty) = 1$. In Generalized Hölder spaces $H(\omega)$ [1, 3] we study the singular integro- differential equations with kernels of Cauchy type (SIDE)

$$\begin{gathered}(Mx \equiv) \sum_{r=0}^{q} [\tilde{A}_r(t) x^{(r)}(t) + \tilde{B}_r(t) \frac{1}{\pi i} \int_\Gamma \frac{x^{(r)}(\tau)}{\tau - t} d\tau + \\ + \frac{1}{2\pi i} \int_\Gamma K_r(t,\tau) \cdot x^{(r)}(\tau) d\tau] = f(t), \quad t \in \Gamma,\end{gathered} \tag{1}$$

where $\tilde{A}_r(t), \tilde{B}_r(t), f(t)$ and $K_r(t,\tau)$ $(r = \overline{0,q})$ are given functions ; $x^{(0)}(t) = x(t)$ is the unknown function; $x^{(r)}(t) = \dfrac{d^r x(t)}{dt^r}$ $(r = \overline{1,q})$; q is a natural number. We search for the approximative solutions of equation (1) in the class of functions, satisfying the condition

$$\frac{1}{2\pi i} \int_\Gamma x(\tau) \tau^{-k-1} d\tau = 0, \quad k = \overline{0, q-1}. \tag{2}$$

We have suggested the numerical schemes of collocation methods for approximative solution of SIDE. The equations are defined on the arbitrary smooth closed contours of complex plane. The collocation methods are based on Fejér points. [2] Theoretical background of collocation methods has been obtained in Generalized Hölder spaces.

Bibliography

[1] A.I. Gusejnov and Ch.S. Muchtarov, *Introduction in the theory of nonlinear singular integral equations*, Nauka, Moscow, 1980, (in Russian).

[2] V.I. Smirnov and N.A. Lebedev, *Functions of a Complex Variable - Constructive Theory*, MIT Press, Cambridge, 1968.

[3] G.A. Zolotarevski, *The approximation of functions in generalized Hölder spaces*, Computer Science Journal of Moldova **3** (1995), no. 9, 300–306.

*This research was supported by Central European University ("CEU") Special and Extension Programs.

†caraush@usm.md

A very simple (but very effective) spline approximation of the Priestley Glacier

L. De Biase[*], G. Fossati, P. Smith, M. Uboldi
Department of Environmental Sciences
University of Milan-Bicocca (Italy)

The Priestley glacier is in the Antarctica Region. A radio echo-sounding sample was collected to single out the bedrock and surface profile of a portion of the glacier. A plane equipped with the proper tools made three passages over a part of the glacier and provided data (longitude, latitude, altitude triplets) along three lines, very close to one another, following a "likely downhill" direction. At two spots no flight could provide data.

Let's analyze each datum $d = ((long, lat), alt)$: the couple $(long, lat)$ reveals the sampling position, while alt is the altitude measured by the echo-sounding. First of all, the three lines made by the couples $(long, lat)$ were replaced by their linear least squares approximation. The $(long, lat)$ data were projected onto the resulting line l and the origin was put at the first point (after a sorting in lexicographic order); the alt coordinates were kept as measured and coupled with the projected points. Subsequently two natural cubic spline functions were determined to describe the glacier bedrock and its surface. The approximation was very good and smooth in the regions were enough data were available, but it presented huge oscillations at the spots with missing data. In order to improve the approximation, new data were reconstructed by BEDMAP and RAMP databases. The approximation was calculated again and its results for the glacier surface were compared to aerial images of the region and proved to be very accurate.

For the transversal sections, following the analysis presented in [1], parabolas were defined on the basis of triplets of points: the two surface points were reconstructed by means of isographic maps, the third point was the bedrock point provided by the spline approximation. On the domain defined by such an approximation, discretized by means of rectangular volumes, the study of mass and energy flows was performed by the method described in [3, 2].

Bibliography

[1] D. Bastoncelli, *Analisi geomorfica quantitativa di forme del rilievo relitte e sepolte per la ricostruzione della storia glaciale cenozoica della Terra Vittoria Settentrionale*, Ph.D. thesis.

[2] S. Ceci, L. De Biase, and G. Fossati, *Heat transfer in 3D water and ice basins*, Advanced Computational Methods in Heat Transfer IX, WIT Transactions on Engineering Sciences, vol. 53, WIT Press, 2006.

[3] A. Deponti, V. Pennati, and L. De Biase, *A fully 3D finite volume method for incompressible Navier-Stokes equations*, Int. J. for Numerical Methods in Fluids (2006).

[*] lucia.debiase@unimib.it

On the use of Kernel-based methods in physical modeling of sounds

C. Drioli[*], D. Rocchesso[†]
Department of Computer Science
University of Verona (Italy)

Kernel based methods have proven to be an effective tool for nonlinear modeling of dynamical systems [3]. In most cases, a black-box approach is used to model observed time series. However, black-box models can be improved when prior information about model structure is available [4]. In this paper we discuss an approach to the modeling of acoustic systems that combines prior information, exploited through physical modeling, and LS-SVM regression [5]. We demonstrate our approach on two case studies, both addressing the broad class of acoustic systems for which the sound generation is obtained through the interaction of a linear system (resonator) and a nonlinear system (excitation). The first case is a physically based impact model[1], were the resonator is described in terms of its normal modes and the nonlinear contact force is modeled through a simplified collision equation and kernel regression. In the second case study, a model of the voice phonation is illustrated in which the vocal cords are represented by a lumped linear mass-spring system and the nonlinear flow component is modeled through simple Bernoulli-based equations and kernel regression [2]. The discussion of the case studies focuses on these fundamental aspects: 1. structural aspects of the numerical models that have to be accounted for in order to merge physical modeling and kernel methods; 2. fitting of the models to real data. It is shown how kernel based regression can be exploited to fit physically based numerical models to observed data 3. computational aspects. The efficiency and computational complexity is discussed for different structural choices (e.g. radial basis functions vs compact support radial basis functions).

Bibliography

[1] F. Avanzini, M. Rath, and D. Rocchesso, *Physically-based audio rendering of contact*, Proc. International Conf. on Multimedia and Expo, vol. 2, 2002, pp. 445–448.

[2] C. Drioli, *A flow waveform-matched low-dimensional glottal model based on physical knowledge*, J. Acoust. Soc. Am. **117** (2005), no. 5, 3184–3195.

[3] S. Elanayar and Y.C. Shin, *Radial basis function neural network for approximation and estimation of nonlinear stochastic dynamic systems*, IEEE Trans. on Neural Networks **5** (1994), no. 4, 594–603.

[4] M. Espinoza, J.A.K. Suykens, and B De Moor, *Kernel based partially linear models and nonlinear identification*, IEEE Trans. on Automatic Control **50** (2005), no. 10, 1602–1606.

[5] J.A.K. Suykens and J. Vandewalle, *Recurrent least squeares support vector machines*, IEEE Trans. on Circuits and Systems-I (2000), no. 7, 1109–1114.

[*]`drioli@sci.univr.it`
[†]`Davide.Rocchesso@univr.it`

Three-pencil lattice in a closed form

G. Jaklič[*], J. Kozak[†], M. Krajnc[‡], V. Vitrih, E. Žagar[§]
Department of Mathematics
and Institute of Mathematics, Physics and Mechanics
University of Ljubljana (Slovenia)
Primorska Institute of Natural Sciences and Technology
University of Primorska, Koper (Slovenia)

In this work, a closed form of a three-pencil lattice is introduced. The explicit representation can be used to obtain lattice points in a simple and numerically stable way and provides a basis for more subtle analysis of bivariate polynomial interpolants.

[*] gasper.jaklic@fmf.uni-lj.si
[†] Jernej.Kozak@fmf.uni-lj.si
[‡] marjetka.krajnc@imfm.uni-lj.si
[§] emil.zagar@fmf.uni-lj.si

An Approach by Vector Extrapolation Methods to the Gummel Map

M.R. Russo[*], R. Bertelle[†]
Department of Pure and Applied Mathematics
University of Padua (Italy)

The numerical approximation of nonlinear partial differential equations requires the computation of large non–linear systems, which are, typically solved by iterative schemes. At each step of the iterative process, a large and sparse linear system has to be solved, and the amount of time spended per step growth with the dimension of the problem. Furthermore, also the number of nonlinear iteration may growth with the dimension of the problem itself. As a consequence, the convergence rate may become very slow, requiring massive cpu–time to compute the solution. In all such cases it is important to improve the rate of convergence of the iterative scheme. This can be achieved by vector extrapolation methods [1, 4].

In this work we apply vector extrapolation methods to the electronic device simulation to improve the rate of convergence of the family of Gummel decoupling algorithms [3], widely used in the iterative solution of the Drift Diffusion system. The semiconductor simulation is a very high growth rate field, because electronic devices manufactures needs a striking low time to market for their new products and so there is the needs of fast simulation algorithms [2, 5].

Preliminary numerical results, and comparison with standard simulations, are presented for basic electronic devices.

Bibliography

[1] C. Brezinski and M. Redivo-Zaglia, *Extrapolation Methods, Theory and Practice*, North–Holland, Amsterdam, 1991.

[2] G.F. Carey, W.B. Richardson, C.S. Reed, and B.J. Mulvaney, *Circuit, device and process simulation*, John Wiley & Sons, U.K., 1996.

[3] H.K. Gummel, *A self-consistent iterative scheme for one dimensional steady state transistor calculations*, IEEE Trans. Electron. Device **11** (1964), 455–465.

[4] K. Jbilou and H. Sadok, *Vector extrapolation methods. Applications and numerical comparison*, J. Comput. Appl. Math. **122** (2000), 149–165.

[5] T. Kerkhoven and Y. Saad, *On acceleration methods for coupled nonlinear elliptic systems*, Numer. Math. **4** (1992), 525–548.

[*] mrrusso@math.unipd.it
[†] bertelle@math.unipd.it

Meshfree Cubature by Radial Basis Functions*

A. Sommariva†, M. Vianello‡
Department of Pure and Applied Mathematics
University of Padua (Italy)

S. De Marchi§
Department of Computer Science
University of Verona (Italy)

We give a survey of some recent results on the construction of numerical cubature formulas from scattered samples of small/moderate size, by using interpolation with RBF (Radial Basis Functions). Error analysis and numerical tests with random points in the unit square have shown that Thin-Plate Splines and Wendland's compactly supported RBF provide stable and reasonably accurate formulas (cf. [2]). The method has then been extended to cubature by RBF over the sphere (cf. [4]), and over arbitary (convex as well as nonconvex and even multiply connected) polygons by Thin-Plate Splines via Green's formula (cf. [3]). We also investigate dependence of the cubature accuracy on nodes location (cf., e.g., [1]), and we present numerical tests and applications.

Bibliography

[1] S. De Marchi, R. Schaback, and H. Wendland, *Near-optimal data-independent point locations for radial basis function interpolation*, Adv. Comput. Math. **23** (2005), no. 3, 317–330.

[2] A. Sommariva and M. Vianello, *Numerical cubature on scattered data by radial basis functions*, Computing **76** (2005), 295–310.

[3] ______, *Meshless cubature by Green's formula*, submitted (preprint UNSW AMR06/10), 2006.

[4] A. Sommariva and R. Womersley, *Integration by RBF over the sphere*, submitted (preprint UNSW AMR05/17), 2005.

*Work supported by the "ex-60%" funds of the Universities of Padua and Verona, by the INdAM-GNCS, and by the ARC MASCOS (research assistantship of A. Sommariva at the School of Mathematics, UNSW Sydney, Australia, 2004-2005).

†alvise@math.unipd.it
‡marcov@math.unipd.it
§demarchi@sci.univr.it

Approximation Methods in Finance

Improved radial basis function methods for multi-dimensional option pricing*

E. Larsson†, U. Pettersson‡, J. Persson§
Department of Information Technology
Uppsala University (Sweden)

K. Åhlander
TietoEnator Digital Innovations, Kista (Sweden)

G. Marcusson
Försäkringsmatematik AB, Stockholm (Sweden)

We have derived a radial basis function (RBF) based method for the pricing of financial contracts by solving the Black–Scholes partial differential equation. As an example of a financial contract that can be priced with this method we have chosen the multi-dimensional European basket call option. We suggest a general approach for how to implement boundary conditions, and which types of boundary conditions to use in more than one dimension. We have shown numerically that our scheme is second order accurate in time and spectrally accurate in space for constant shape parameter. For other, non-optimal choices of shape parameter values, the resulting convergence rate is algebraic. We propose an adaptive node point placement that improves the accuracy compared with a uniform distribution. Compared with an adaptive finite difference method, the RBF method is 20–40 times faster in one and two space dimensions and has approximately the same memory requirements.

We also explore a different approach, were the problem is transformed to a symmetry respecting form and the generalized Fourier transform is employed for block diagonalization of the RBF approximation matrices. Time and memory requirements are reduced by a constant factor, but this factor grows with the number of dimensions.

*The work was supported by FMB, the Swedish Graduate School in Mathematics and Computing and by a grant from the Swedish Research Council.

†bette@it.uu.se
‡ulrikap@it.uu.se
§jonasp@it.uu.se

Using lattice rules to solve high-dimensional integration problems from mathematical finance*

B. Waterhouse†
School of Mathematics and Statistics
University of New South Wales, Sidney (Australia)

In recent years there have been many significant advances in the field of quadrature rules for numerical integration. For high-dimensional problems, that is, for problems where the dimension is in the hundreds or thousands, the Monte Carlo technique is the most commonly used. The Monte Carlo method gives an n-point approximation to the d-dimensional integration problem

$$\int_{[0,1]^d} f(\mathbf{x})\mathrm{d}\mathbf{x} \approx \frac{1}{n}\sum_{k=1}^{n} f(\mathbf{x}_k)$$

where the points $\mathbf{x}_k \in [0,1]^d$ are chosen randomly. This is a powerful technique because the error converges with order $O(n^{-1/2})$. It is important to note that this convergence is independent of the dimension.

It is well-known that there exist choices of point sets $\mathbf{x}_k \in [0,1]^d$ such that the error converges with order $O(n^{-1})$. That is, we make some sensible choice of the points $\mathbf{x}_k$ rather than choosing them randomly as in the Monte Carlo method. One of these techniques is known as lattice rules. This talk will give an introduction into the use of lattice rules to solve high dimensional integration problems.

We will be examples of the use of lattice rules to solve practical problems of mathematical finance where the dimension of the problem is in the tens of thousands.

*Work supported by the Australian Research Council under its Centres of Excellence Program
†benjw@maths.unsw.edu.au