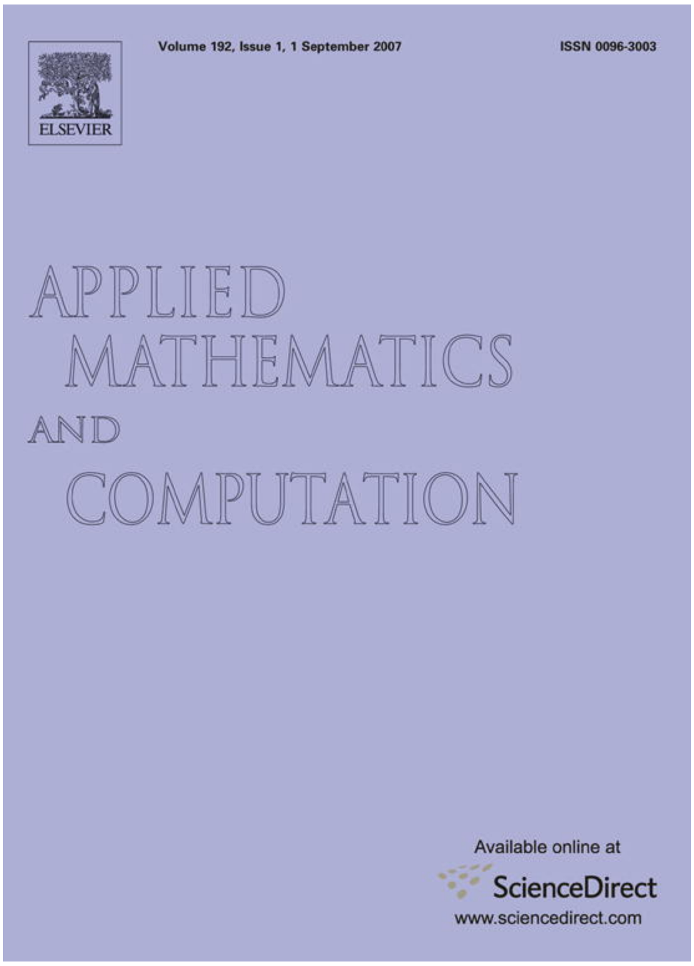
Volume 192, Issue 1, 1 September 2007
ISSN 0096-3003
ELSEVIER
APPLIED MATHEMATICS AND COMPUTATION
Available online at
ScienceDirect
www.sciencedirect.com

Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics and Computation 192 (2007) 180–190
APPLIED MATHEMATICS AND COMPUTATION
www.elsevier.com/locate/amc

Mathematics and wine

Stefano De Marchi

Department of Computer Science, University of Verona, 37134 Verona, Italy

Abstract

The aim of the paper is twofold. Firstly, to show that mathematics can also be used to describe many facts and aspects related to wine and especially wine tasting. Secondly, we would like to show that the wine is a chaotic dynamical system that, thanks to mathematics, can be properly studied. This can help wine makers and wine tasters to understand better.

Keywords: Applied mathematics; Wine tasting; French paradox

1. Introduction

The ‘‘leitmotif’’ of this paper is essentially the importance of mathematics in all aspects of real life and also in many aspects connected to the most important beverage of the *ancient world*, wine. Thanks to the various applications of mathematics, people have started to think of mathematics not only as a arid topic for *‘‘strange’’* persons called mathematicians, but as a basic and fundamental tool that everyone should understand, as much as they can, because mathematics can help to model almost everything.

The one who is writing this paper is a mathematician that for fun one day decided to learn more about wine tasting and took the course for becoming a sommelier. A sommelier judges a wine following the so called *sensorial analysis* which from the mathematical point of view is an algorithm and in fact, on studying the official books of the Italian Sommeliers Association [1], one can see many ways of modeling the wine tasting in a mathematical framework.

The paper is indeed an attempt to show the interesting connections between wine tasting and some related problems with mathematics. In the next Section we start with two simple problems whose mathematical solutions show at a glance the importance of mathematics also in enology. In Section 3 we discuss an interesting problem known as the *French paradox*. After modeling the problem in a probabilistic way, we provide an analytical solution of it. Then, in Section 4, we move to the various steps of wine tasting finding interesting connections with geometry and analysis. Wine, mathematically speaking, is a dynamical system that has its initial time corresponding to the production of the must, then after the alcoholic fermentation it becomes a wine and depending on its characteristics we can age it in ‘‘barriques’’ or in bottles. This is mainly what we show in Section 5. Moreover, the ‘‘system-wine’’ depends on many variables, part of them come from the environment,

E-mail address: stefano.demarchi@univr.it

0096-3003/$ - see front matter
doi:10.1016/j.amc.2007.02.150

especially soil and climate, and the others from the grapes, that combine in chaotic way. This is the intuitive reason why the wine can be considered a *chaotic dynamical system*. We conclude by suggesting a particular wine called *Chaos*, a mix of montelpulciano, syrah and merlot. The wine is produced in the region Marche in Italy and its label is the Mandelbrot set. *Chaos* was in some sense the inspiration of this paper.

2. Two simple problems

Our tour in the subject *mathematics and wine* starts with two mathematical diversions showing that mathematics can be useful to solve some interesting problems related also to wine.

2.1. The problem of the three barrels

Problem 1. A man has three barrels, one completely full with 8 hectoliters (hl.) of wine and the other two empty of 5 and 3 hl., respectively (see Fig. 1). The question is: how can the man divide equally the wine with the help only of the empty barrels?

The solution runs as follows. At the beginning the barrels contains 8, 0, 0 hl. respectively. If we number them as 1,2,3, then to get the solution we may proceed as follows.

1. Pour the wine from 1 to 3: we get 5 0 3 hl.
2. Pour the wine from 3 to 2: we get 5 3 0 hl.
3. Pour the wine from 1 to 3: we get 2 3 3 hl.
4. Pour the wine from 3 to 2: we get 2 5 1 hl.
5. Pour the wine from 2 to 1: we get 7 0 1 hl.
6. Pour the wine from 3 to 2: we get 7 1 0 hl.
7. Pour the wine from 1 to 3: we get 4 1 3 hl.
8. Pour the wine from 3 to 2: we get 4 4 0 hl.

The answer is simply an algorithm that dates back to Niccolò Tartaglia in his book number 16 probably written in 1560. A question for some clever readers is: *does there exit a solution with less than 8 steps?*. The answer is "yes, one exists", but we leave it as an exercise.

2.2. The problem of the glasses of water and wine

Problem 2. Take two glasses, one with water and one with wine and transfer a certain quantity of water into the wine, then the same quantity of the mixture (wine and water) is transferred into the glass of water. The question is: do we have more water in the glass of wine or wine in the glass of water?

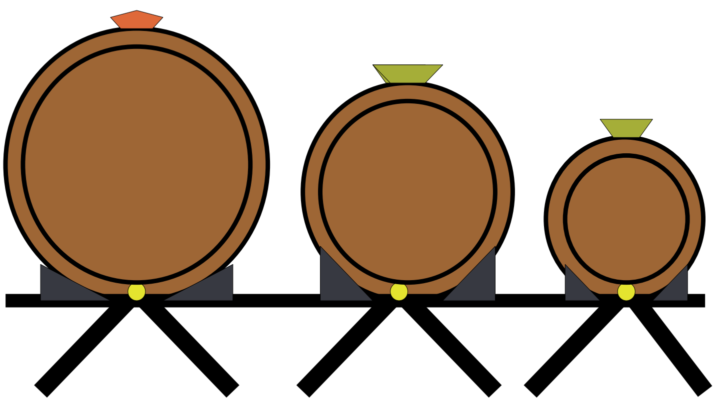

Fig. 1. The three barrels: the leftmost is the full one (see the closed cork).

The well-known and simple answer is that the quantities are the same. This is due to the fact that if a quantity x of liquid is taken from one glass a similar quantity x goes back to the other glass (see e.g. [5, p. 279]).

The solution to this problem has a logical bug: we are not considering the physical characteristics of the water and the wine. In fact, when we take from a mixture of two fluids a certain quantity, the quantity of the single compound in the mixture is different from its quantity in the mixture. The deviation of these two quantities is of order $\sqrt{n}$, with n the number of moles that we assume to be present in the liquid. Hence, we probably have the same quantities, as claimed in [5, p. 279], if this deviation in the liquid will be reduced to $\pm\sqrt{n}$ and it seems possible with 47 mutual exchanges (cf. [5, p. 287]).

3. The French paradox: an estimate

The French paradox refers to the fact that people in France suffer a relatively low incidence of coronary heart disease, despite their diet being rich in saturated fats. The phenomenon was first noted by the Irish physician Samuel Black in 1819. In 1991 the French epidemiologist Serge Renaud, in his famous interview during the program *60 Minutes* on the American network CBS, speaking about the connection between pathology of coronaries and the assumption of lipids, he introduced the name *French paradox* or *Bordeaux effect* (see also [7,8]). As a remark, we recall that Prof. Renaud, for his studies on this field, has been awarded in 2005 by the President Chirac of the *Légion d'Honor* which represents the most important award and acknowledgment given to outstanding people in France.

In simple words the paradox can be stated as follows:

PARADOX. 1 The drinking especially of red wine, determines a significant reduction in the risk of cardiovascular diseases even though the diet is rich in lipids.

After Renaud's studies, we now know that this effect in mostly due to one important substance present in the wine called *reseveratrol*. Reseveratrol is a chemical found in wine grape skins and is a form of estrogen called phytoestrogen, a hormone known to protect against heart disease.

In what follows we propose an estimate of the quantity of daily red wine that we may drink in order to preserve ourselves from cardiovascular diseases but avoiding liver disease.

Let us assume that the wine behaves like a "population-wine" that evolves in time as a *Malthus model*. Thus, the probability p to get a cardio-circulative disease drinking x liters per day, decreases exponentially by the formula

$$p(x) = p_0 \mathrm{e}^{-\frac{x}{x_p}}, \tag{1}$$

where p_0 is a constant representing the probability that such disease decreases for an abstainer and x_p the reciprocal growth rate of the "population-wine". In fact, the Eq. (1) is the solution of the Cauchy problem

$$p'(x) = -\frac{1}{x_p} p(x), \quad x \geqslant 0,$$
$$p(0) = p_0,$$

that represents the Malthus model.

On the other hand, the probability of an `increase` of the same disease can be expressed similarly by

$$q(x) = q_0 \mathrm{e}^{\frac{x}{x_q}}, \tag{2}$$

where q_0 is a constant representing the probability for an abstainer to catch the disease and x_q the corresponding growth rate for this setting.

To get an estimate we sum up both probabilities, that is Eqs. (1) and (2), obtaining

$$r(x) = p_0 \mathrm{e}^{-\frac{x}{x_p}} + q_0 \mathrm{e}^{\frac{x}{x_q}}. \tag{3}$$

Our aim is to find the extremal points of $r(x)$. Hence

$$r'(x) = -\frac{1}{x_p} p_0 \mathrm{e}^{-\frac{x}{x_p}} + \frac{1}{x_q} q_0 \mathrm{e}^{\frac{x}{x_q}} = 0.$$

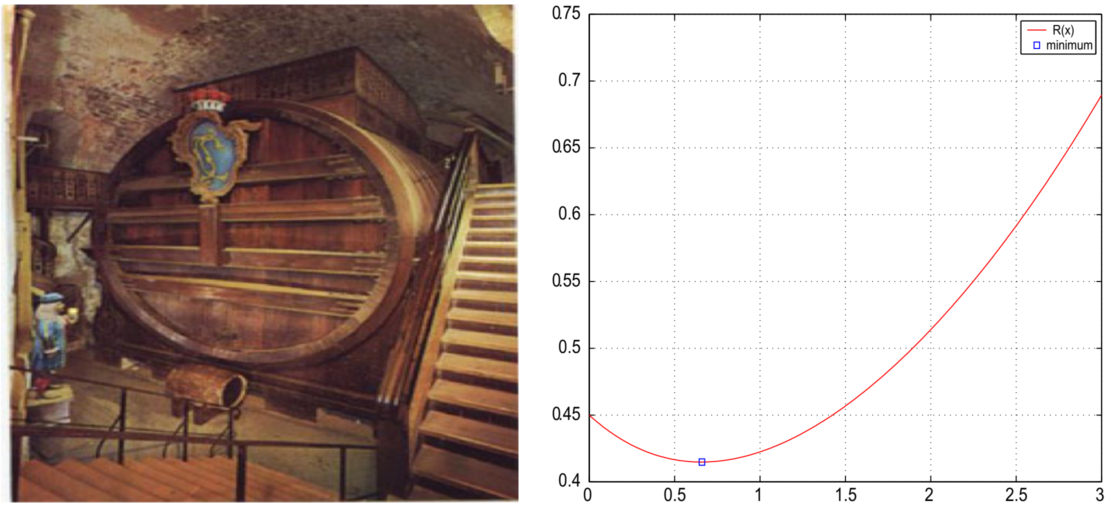

Fig. 2. *Left:* the huge Heidelberg barrel. Its dimensions are: 7 m. height and 8.5 m. width. It contains 221,726 l. Over the barrel there is room for a ball stage. *Right:* the function $r(x)$ with the choice of parameters as in Remark 1.

The only point is

$$x^* = \left(\frac{x_p x_q}{x_p + x_q}\right)\left[\log\left(\frac{x_q}{x_p}\right) + \log\left(\frac{p_0}{q_0}\right)\right]. \tag{4}$$

Note that $r(0) = p_0 + q_0 > 0, \lim_{x\to+\infty} r(x) = +\infty, r'(x)$ is negative for $0 \leqslant x < x^*$ and positive for $x > x^*$, thus x^* is the *unique* point at which $r(x)$ attains its minimum value and so $r(x^*)$ is its global minimum.

Remark 1. The value x^* depends on the quantities p_0, q_0, x_p, x_q. Not all choices make sense. For instance in [8] the authors proposed these (reasonable) choices.

- $x_p = 1$ and $x_q = 3$. The value for x_p comes from usual habit while $x_q = 3$ is related to the story of the dwarf Percheo and the hugest barrel in the world of the Heidelberg castle (see Fig. 2, left). The inhabitants of the castle used to drink at least two liters of wine per day while the dwarf Percheo twelve bottles (about 6 l). He was very healthy till the day he drank, for a lost bet, two glasses of water and suddenly he died ... maybe because the water, at those time, was polluted.
- $p_0 \sim q_0$. This is a reasonable choice since both p_0 and q_0 represent the probability to get or to avoid any such disease by an abstainer.

For the choice $x_p = 1,\ x_q = 3,\ p_0 = 0.2,\ q_0 = 0.25$ we have the solution $x^* \approx 0.76$ l (see Fig. 2, right) which suggests that one bottle per day of red wine (possibly drunk during meals!) is the suitable quantity that prevents from cardiovascular diseases while avoiding any liver disease.

Apart from the resveratrol, in the wine there are many other compounds (at least 600 are very significant) and the drinking of a bottle of wine per day, as obtained above, if for someone can really prevent from cardiovascular diseases for the majority of people this will lead to other physical problems. That is why this is a paradox. On the other hand, the properties of resveratrol are well studied and nowadays there are medicines based on the chemical components of resveratrol used to prevent the flu.

4. The mathematics of wine tasting

In this section we try to present various aspects of wine tasting technique, used by experts like sommeliers and enologists, that can be defined mathematically. In particular, when a wine is tasted, in order to understand its characteristics there are *essentially three steps* or analyses that must be done. The first one is the so-called *visual analysis* of the wine, in which one judge the color, the limpidity, the transparency and, in the case of sparkling wines, the consistency, the number and the persistence of bubbles. A second check is the so-called *olfactory examination* where the experts should *objectively* make an analysis of the olfactory intensity, olfactory persistence and finding some correspondence with known scents and smells. The more scents one can clearly identify the more complex will be the wine. The third and essentially the last important aspect to check is the *gustative analysis*. Here one has to analyze the so-called soft and hard parts of a wine. The soft part is made up by the sugars, the alcohols and the poly-alcohols while the hard part, is made up by the acids, the tannins and the mineral salts. Finally, on the basis of the previous analysis one can say that a wine has a general structure which is thin up to vigorous and also say something about its harmony and evolution. For more details see, e.g., the Ref. [1].

It is not the aim of this paper to write a treatise on the techniques of wine tasting, but this small introduction can help us in understanding some of the considerations we are describing.

4.1. The geometry of the olfactory examination

In the olfactory examination one checks firstly two important features of the wine: the intensity and the complexity of the wine scents. The intensity represents the *height* of the scents while the complexity is their persistency in time (i.e. its duration), that is their *length*. The sum of these two characteristics is the *olfactory quality* of our wine. Then, the more intense and complex the wine, the better is its quality.

Therefore, if we use a cartesian plane $(x, y) = (Complexity, Intensity)$ the olfactory quality, $q(x, y)$ is an increasing two-dimensional function. Obviously $q(x, y)$ will not increase with the same gradient along x and y because, for instance, the intensity can be greater than the complexity. In Fig. 3 we show a possible mathematical representation of the quality of a wine as the monotone function $q(x, y) = x^4 + y^2$. For this function we may say that the wine is more complex than intense.

Another aspect related to the olfactory complexity is the *number* and *types* of scents that one can identify in smelling the wine. These scents are usually grouped in three sets: *primaries* strictly connected to the particular kind of grapes and the environment, *secondaries* formed essentially during the wine fermentation and *tertiaries* due to the evolution and ageing processes. The union of these three scents makes the wine olfactory complexity and the experts afterwards will say that a wine is `deficient`, `poorly complex`, `quite complex`, `complex`, `wide`.

Let $\mathcal{O}_c$ be the olfactory complexity. Thus, $\mathcal{O}_c$ is the function whose domain is the set

$$D = P \cup S \cup T$$

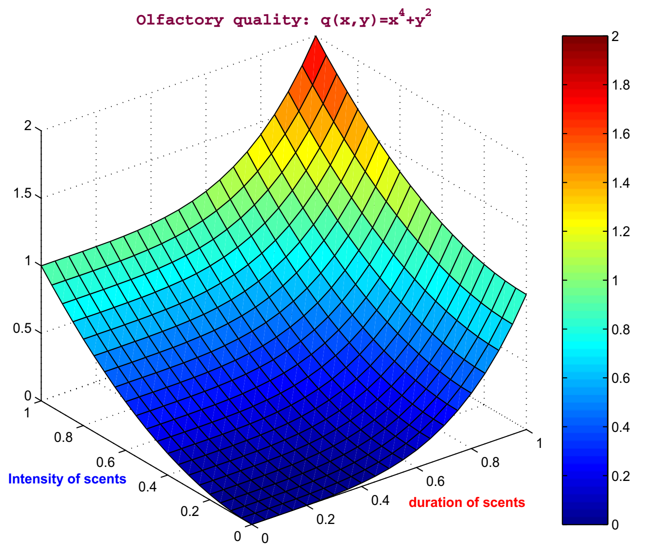

Fig. 3. A mathematical representation of the complexity as the monotone function $q(x, y) = x^4 + y^2$.

consisting of the union of the sets representing the primary, secondary and tertiary scents respectively, with values in the discrete set

$$A = \{\texttt{deficient}, \texttt{poorlycomplex}, \texttt{quitecomplex}, \texttt{complex}, \texttt{wide}\}.$$

Remark. This *functional* way to describe the olfactory complexity, can be applied to almost all other aspects of wine tasting, such as in the description of the acidity or the alcohol (cf. [1]). In fact, sommeliers in describing a wine associate to any of the wine characteristics a scale of values. These scales represent indeed the images of the corresponding functions.

4.2. Wideness, length and poly-alcohols of wines

These characteristics are related to the evolution of wines (cf. [10]).

Definition 1. We say that a wine is `wide`, when its visual, olfactive and gustative sensations are very intense, nearly *explosive*, but limited in time.

Therefore the wideness of wine is a function,

$$\mathscr{W} : D_{\mathscr{W}} \subset \mathbb{R}_+ \to \mathbb{R}^3.$$

which has a very high derivative in each direction. The (temporal) support $D_{\mathscr{W}}$ can be considered as a small interval of $\mathbb{R}_+$.

On the other hand, the `length` of a wine expresses slow changes in time.

Definition 2. We say that a wine is `long`, when its visual, olfactive and gustative sensations show off slowly in time.

Then, the length of wine is a function,

$$\mathscr{L} : D_{\mathscr{L}} \subset \mathbb{R}_+ \to \mathbb{R}^3.$$

which has slowly growing derivatives in each direction. The (temporal) support $D_{\mathscr{L}}$ can be considered as all $\mathbb{R}_+$.

A third aspect, which introduces us to the *evolutionary system* representing a wine, is found in the gustative analysis, that is the poly-alcohols. These are the most important compounds of a wine and are fundamental for its structure, in particular they represent the soft part of the structure. They are mostly due to the glycerine content. A wine with respect to poly-alcohols can be *sharp, scarcely soft, quite soft, soft* and *velvety*. In mathematics, these characteristics can be represented, or better, modeled by functions of increasing regularity, positive and "bell-shaped" like, for instance, polynomial *splines*. Polynomial splines, or simply splines, are piecewise continuous polynomials connected with the highest possible degree of continuity at the connection points. The classic cubic spline, which is of order +4, is globally $\mathscr{C}^2$. In general splines of degree k, order $k+1$, are globally $k-1$ continuous. Splines have also a stable basis formed by the B-splines or *Basic*-splines. In Fig. 4, we plot five *B-splines* of orders +2 up to +6 on the unit interval, which are $\mathscr{C}^s, s = 1, \ldots, 5$ respectively, and that can be seen as an original mathematical representation of the corresponding scale of poly-alcohols in a wine. The readers interested to splines functions can refer, for example, to the fundamental book by Carl de Boor [2]. An example of wines which are *simply continuous*, we mention the Italian *Verdicchio* or the German *Riesling* ("trocken" type). For wines which are very smooth, i.e., velvety, we may recall the French *Sauterns* or the Hungarian *Tokaij Aszu*.

5. The mathematics in wine fermentation and wine ageing

The intuitive idea that *a wine is a dynamical system* is the main concern of this Section. The adjective *dynamical* means something that evolves with respect to time, that is why the theory of dynamical systems is sometimes referred to as the *mathematics of time*. The dynamical system concept is a mathematical formalization

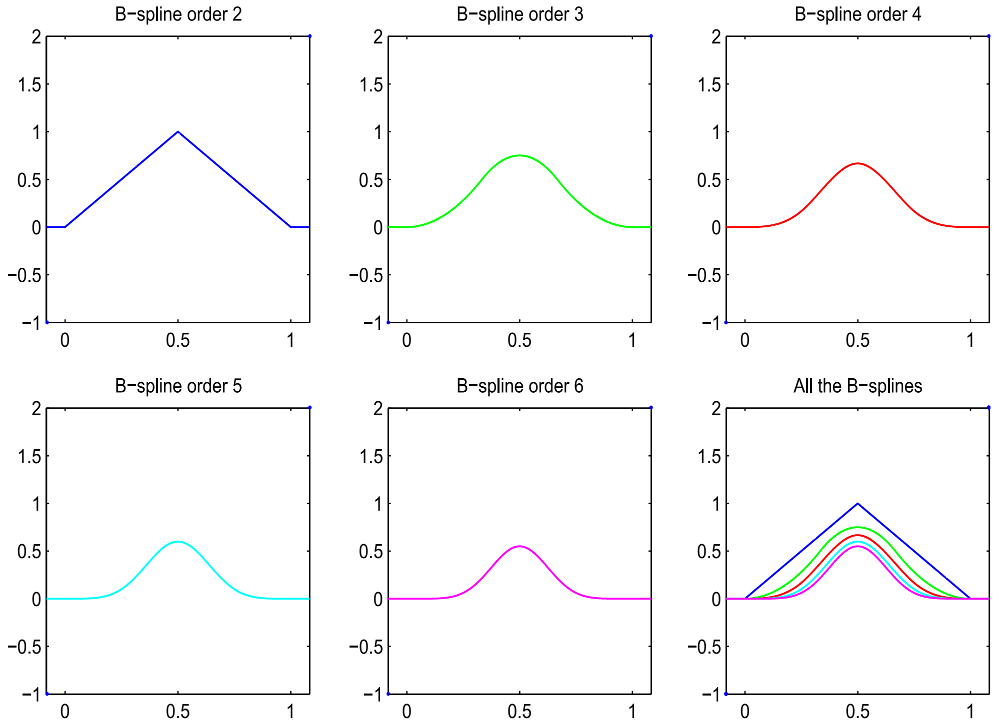

Fig. 4. B-splines of orders 2 up to 6 on the unit interval.

for any fixed "rule" which describes the time dependence of a point's position in its ambient space. Examples are: the mathematical models used to describe the swinging of a clock pendulum; the flow of water in a pipe; the number of fish each spring in a lake or the evolution and ageing of wine. The evolution of wine due to the ageing is treated, for example, in the papers [4,9]. In [4] the authors simply tried to model the changes of oak-related compounds by studying the diffusion kinetics of some special compounds (lactones, guaiacol and 4-methylguaiacol, and vanillin) in wine maturation. They showed that when the barrels are new the diffusion kinetics measured in terms of the *rate of accumulation* of the compound, can be fit by exponentials functions of the rate, otherwise, when the barrels are already used, by polynomials of decreasing degree. In [9] the study of a *Fickian diffusion model* for simulating the wine losses during ageing in oak barrels, has been presented. This model is based on *Fick's second law* that models non-steady state diffusion processes, that is processes in which the concentration within the diffusion volume changes with respect to time. Letting $u(x,t)$ the concentration function (subject to some Dirichlet and Robins boundary conditions) the diffusion equation corresponding to Fick's second law is

$$u_t(x,t) = Du_{x,x}(x,t), \tag{5}$$

where D is the diffusion coefficient which depends on the type of wood.

The interest of these two examples is the fact that the wine and the phenomena connected to the ageing is dynamically evolving as a diffusion process.

So far we have not yet discussed what is a dynamical system and in particular a chaotic dynamical system: this is what we are going to say.

To understand a dynamical system we need to know its *state*. The state of a dynamical system is determined by a collection of real numbers, or more generally by a set of points in an appropriate state space.

To be clearer, for modeling the dynamics of a system, we start from a set $x_1, \ldots, x_n$ of measurable quantities that represent the system's state at the time t, that is

$$\mathbf{x}(t) := (x_1(t), \ldots, x_n(t)).$$

If t is a real number, the dynamical system is called *continuous* otherwise, when t is a natural number, the system is called *discrete*.

The evolution of system is formally a function, that once the initial state $\mathbf{x}_0$ at t_0 is known, allows to uniquely determine the state of the system in any successive time t:

$$\mathbf{x}(t) = \mathscr{F}(t_0, \mathbf{x}_0; t). \tag{6}$$

Hence, starting from the initial condition $\mathbf{x}_0(t_0)$, the set of values obtained by (6) is the *trajectory* of the system passing through $\mathbf{x}_0(t_0)$. In practise, it is not easy to find the operator $\mathscr{F}$ and people try to recover it by using some differential equations representing the *local evolution* of the system. These equations, in the continuous setting, describe the variation of each state variable x_i w.r.t. the others and itself, too. That is

$$\frac{dx_i}{dt} = f_i(x_1, \ldots, x_n), \quad i = 1, \ldots, n.$$

Unfortunately only in simple cases, for example with linear differential equations, one can find the analytic solution of the system satisfying the initial conditions. Similar considerations can be done for *discrete* dynamical systems where difference equations will be considered instead of differential equations

$$x_i(t+1) = f_i(\mathbf{x}(t)).$$

It is beyond the aim of this paper to go further in the theory of dynamical systems. What we only want to understand is why wine is a *chaotic* dynamical system.

For a dynamical system to be classified as *chaotic*, most scientists will agree that it must have the following properties.

- It must be *sensitive* to initial conditions. That is an arbitrarily small perturbation of the current trajectory may lead to significantly different future behavior. Sensitivity to initial conditions is often confused with chaos in popular accounts.
- It must be *topologically mixing*, in the sense that the system will evolve over time so that any given region or open set of its phase space will eventually overlap with any other given region. Here, *mixing* is really meant to correspond to the standard intuition: the mixing of colored dyes or fluids is an example of a chaotic system.
- Its *periodic orbits* must be *dense*.

A simple example of a real chaotic system is *the smoke of cigarettes*. In fact, even if cigarettes are lighted in macroscopical similar conditions, their smoke can behave in very different ways, depending on the air pressure, the air currents, the air temperature and so on.

Similarly, the fermentation of the must, that contains about 2000 known compounds, depends on the air pressure, humidity, temperature, the lunar phase, and so on. Therefore, small changes in these during the fermentation could influence significantly the production and also the evolution of the wine, so that we can get a *good wine* or a *"good vinegar"*. Fig. 5 shows these two chaotic dynamical systems: the match's smoke and the must. In many physical and mathematical models of wine fermentation kinetics, people mostly study the diffusion of some compounds and their effect on other compounds, such as yeast cells and sugars, or the diffusion of the flavors in oak barrels or due to oak chips. These models are essentially a description of chemical phenomena by means of biological mathematical models. As detailed in [12] the fermentation kinetics model can be subdivided into three parts: a growth model, a substrate model and a product model. Recently the *sigmoidal logistic model* has been one of the most popular used for simulations due to its property of *good fit* of experimental data. In its standard form the sigmoidal curve is the solution of the Cauchy problem

$$\frac{\mathrm{d}p}{\mathrm{d}t} = p(1-p), \quad p(0) = p_0. \tag{7}$$

This first order non-linear differential Eq. (7) is indeed a special case of the well-known *Verhulst logistic model*

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \kappa p\left(1 - \frac{p}{C}\right), \quad p(0) = p_0. \tag{8}$$

where the constant κ represents the growth rate and C the carrying capacity of the system. This model, in the discrete case, is usually represented (after a scaling process) by the iterative map

Fig. 5. (Right) A match's smoke and (Left) the fermenting must.

$$x_{n+1} = \sigma x_n(1 - x_n), \quad n \geqslant 0,$$

which, for values of the parameter σ grater than 3.45, shows some chaotic behavior that graphically produces the well-known *bifurcation diagram* (see Fig. 6, right).

Some dynamical systems are chaotic everywhere but in many cases chaotic behavior is found only in a subset of the phase space. The cases of great interest arise when the chaotic behavior takes place on an *attractor*, since then a large set of initial conditions will lead to orbits that converge to this chaotic region. While most of the motion types mentioned above give rise to very simple attractors, such as points and circle-like curves called *limit cycles*, chaotic motion gives rise to what are known as *strange attractors*, i.e. attractors that can have great detail and complexity. For instance, a simple three-dimensional model of the Lorenz weather system gives rise to the famous *Lorenz attractor* (see Fig. 6) well-known for its butterfly shape. Strange attractors are also *fractals* whose most important representatives are the *Mandelbrot set* and the *Julia set*, famous also as screen savers.

In conclusion, wine fermentation and wine ageing are not simple dynamical systems to study and the study of the wine fermentation should be better modeled by reaction–diffusion equations of the form

$$u_t(\mathbf{x}, t) = D\Delta u(\mathbf{x}, t) + Kg(u(\mathbf{x}, t)), \tag{9}$$

where the function g represent the reaction of the system, usually a non-linear function of u, with the function u depending on the vector $\mathbf{x}$, which represents the variables involved in the evolution, and the time t. For these equations the solution is usually found by sophisticated numerical methods (cf. e.g. [6]). Unfortunately the great number of substances that make up wine, make its behavior often chaotic and therefore people solving numerically the reaction–diffusion equations of the wine evolution, should study other aspects connected to

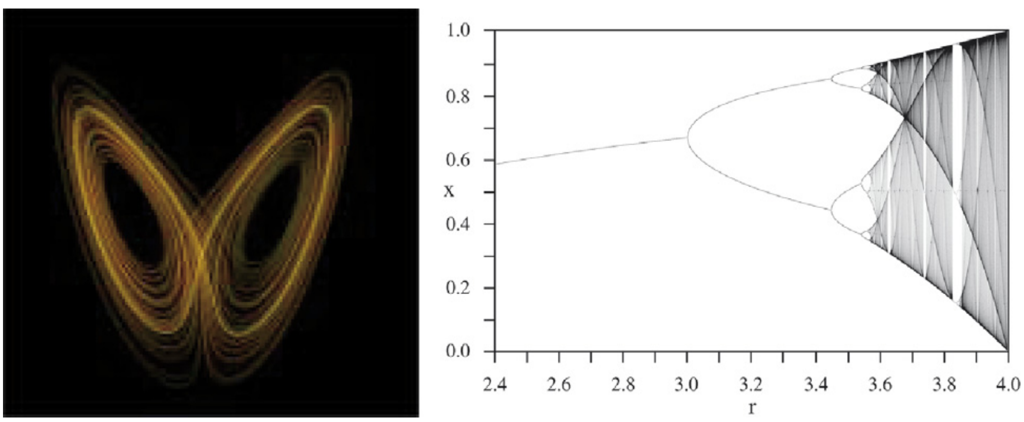

Fig. 6. (Left) Lorenz attractor, (Right) the Logist map and its bifurcation diagram.

Fig. 7. Chaos 1997, 1998 and 1999.

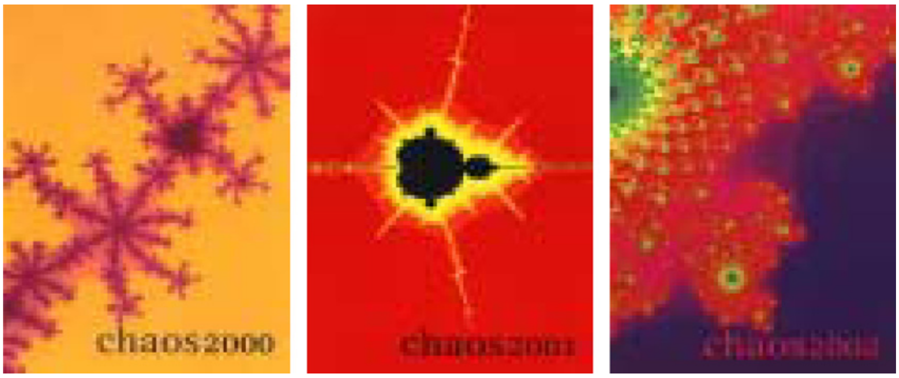

Fig. 8. Chaos 2000, 2001 and 2002.

the numerical solution, such as stability. It will be the aim of a second paper to provide some stable numerical methods for wine fermentation and ageing.

6. "Chaos" is also a wine

We have just seen that wine is composed of at least 2000 compounds that combine and mix together and evolve in time, giving to the wine its "personality". An Italian wine producer, the firm *"Fattoria Le Terrazze"* whose vineyards are in Numana nearby Ancona, has decided since 1997 to call one of its red wines with the name *Chaos* (see also [11]). The producer, on the basis of our previous discussion about the *chaos into the wine*, decided to use as label of Chaos, views of the *Mandelbrot set*. In fact, the Mandelbrot set is a *mathematical object* that for many reasons is considered the prototype of chaos in dynamics. In Figs. 7 and 8 we show all the labels of Chaos.

7. Conclusions

In this paper, we gave an essay of how mathematics can be useful to describe some process connected to wine tasting and its evolution. But this is not all. Mathematics, for instance, is fundamental to understand why champagnes, and in general sparking wines, have bubbles. Of bubbles we would like to understand their birth from the so called *nucleation centers* situated along the surface of the glass; their growth due to the difference of pressure in the glass and the outside air; the speed they reach in rising from their birth to their death, and finally how bubbles die at contact with the air. Mathematics is indeed the perfect tool to simulate all these processes (see the interesting monograph [3]).

Mathematics can also help in optimizing the distribution of vines in a given area. Moreover we can use mathematics in describing the geometry of a bottle and in the design of new bottles and glasses (cf. [10]). Many other aspects, connected to the vine and wine, are better understandable with the help of mathematics. As we said in the Introduction, mathematics is everywhere...also in the wine.

Acknowledgements

I would like to give a special thanks to the Italian Association of Sommeliers, delegation of Padua, who invited me to give this lecture on November 23rd, 2006. Of course I can not forget the wine Chaos 2001 and its producer, Antonio Terni, that we tasted in that unforgettable evening. Special thanks also to Len Bos and Carl de Boor for proofreading the paper.

References

[1] Associazione Italiana Sommeliers, Tecnica della Degustazione, 1B. Ed. AIS (2001).
[2] Carl de Boor, A practical guide to splines, Rev. ed., Applied Mathematical Sciences, 27, Ed. Springer, 2001.
[3] Gérard Liger-Belair, Bollicine, Ed. Einaudi, 2005.
[4] Encarna Gómez-Plaza et al., The effect of successive uses of oak barrels on the extraction of oak-related volatile compounds from wine, Int. J. Food Sci. Tech. 39 (2004) 1069–1078.
[5] M. Gardner, Enigmi e giochi matematici, SuperBur Saggi, 2005.
[6] J.C. Jorge, B. Bujanda, Numerical methods for evolutionary reaction–diffusion problems with nonlinear reaction terms, J. Comput. Appl. Math. 166 (1) (2004) 167–180.
[7] D. Oltolini, La Fisica nel bicchiere, Il sommelier italiano, January 2005.
[8] A. Rigamonti, A.A. Varlamov, Nunc est Bibendum: Divertissment di Fisici attorno a bicchieri di vino, Il Nuovo Saggiatore, January 2004.
[9] A. Ruiz de Adana, L.M. Lopez, J.M. Sala, A Fickian model for calculating wine losses from oak casks depending on conditions in ageing facilities, Appl. Therm. Eng. 25 (2005) 709–718.
[10] M. Soster, Geometria enologica, Organizzazione Interprofessionale per la Comunicazione delle Conoscenze in Enologia, www.oicce.it/sito/ot/ot23/ot23presidente.html.
[11] A. Terni, Matematica e vino, in: Matematica e cultura, Springer, 2006, pp. 241–244.
[12] D. Wang et al., Fermentation kinetics of different sugars by apple wine yeast *Saccharomyces cerevisiae*, J. Inst. Brewering 110 (4) (2004) 340–346.