Some mathematics in the wine
Part I

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Introduction

The "leitmotif" of this paper is essentially the importance of mathematics in all aspects of real life and also in many aspects connected to the most important beverage of the ancient world: wine. Thanks to the various applications of mathematics, people have started to think of mathematics not only as an arid topic for "strange" persons called mathematicians, but as a basic and fundamental tool that everyone should understand, as much as they can, because mathematics can help to model almost everything.

The one who is writing this paper is a mathematician that for fun one day decided to learn more about wine tasting and took the course for becoming a sommelier. A sommelier judges a wine following the so called sensorial analysis which from the mathematical point of view is an algorithm and in fact, on studying the official books of the Italian Sommeliers Association [1], one can see many ways of modeling the wine tasting in a mathematical framework.

The paper is indeed an attempt to show the interesting connections between wine tasting and some related problems, with mathematics. In the next Section we discuss an interesting problem known as the French paradox. After modeling the problem in a probabilistic way, we provide an analytical solution of it. Wine, mathematically speaking, is a dynamical system that has its initial time corresponding to the production of the must, then after the alcoholic fermentation it becomes a wine and depending on its characteristics we can age it in "barriques" or in bottles. This is mainly what we show in Section 3 with emphasis to the mathematics of wine tasting.

The French paradox

The French paradox refers to the fact that people in France suffer a relatively low incidence of coronary heart disease, despite their diet being rich in saturated fats. The phenomenon was first noted by the Irish physician Samuel Black in 1819. In 1991 the French epidemiologist Serge Renaud, in his famous interview during the program 60 Minutes on the American network CBS, speaking about the connection between the Irish physician Samuel Black in 1819. In 1991 the French epidemiologist Serge Renaud, in his non alcoholic beverage called "resveratrol," which represents the most important beverage of the ancient world: wine.

Remark 1

The value \( x^* \) depends on the quantities \( p_0, q_0, x_p, x_q \). Not all choices make sense. For instance in [3] the authors proposed these (reasonable) choices.

\[ x_p = 1 \text{ and } x_q = 3. \]

The value for \( x_q \) comes from usual habit while \( x_p = 3 \) is related to the story of the dwarf Percheo and the hugest barrel in the world of the Heidelberg castle (see Fig. 1, Left). The inhabitants of the castle used to drink at least two liters of wine per day while the dwarf Percheo twelve bottles (about 6 liters) which suggests that \( x_q = 3 \) liters (see Fig. 1, Right) which suggests that one bottle per day of red wine (possibly drunk during meals!)

We now propose an estimate of the quantity of daily red wine that we may drink in order to preserve ourselves from cardiovascular diseases but avoiding liver disease. Let us assume that the wine behaves like a "population-wine" that evolves in time as a Malthus model. Thus, the probability \( p \) to get a cardiovascular disease drinking \( x \) liters per day, decreases exponentially by the formula

\[ p(x) = p_0 e^{-x/p_0} \] (1)

where \( p_0 \) is a constant representing the probability that such disease decreases for an abstainer and \( x_p \) the reciprocal growth rate of the "population-wine". In fact, the equation (1) is the solution of the Cauchy problem

\[ p'(x) = L \left[ e^{-x/p_0} \right], \quad x \geq 0 \]

where \( L \) is the corresponding growth rate of the "population-wine". In fact, the equation (1) is the solution of the Cauchy problem

\[ p'(x) = L \left[ e^{-x/p_0} \right], \quad x \geq 0 \]

that represents the Malthus model.

On the contrary, the probability of an increase of the same disease can be expressed similarly by

\[ q(x) = q_0 e^{-x/q_0} \] (2)

where \( q_0 \) is a constant representing the probability for an abstainer to catch the disease and \( x_q \) the corresponding growth rate for this setting.

To get an estimate we sum up both probabilities, that is equations (1) and (2), obtaining

\[ r(x) = p_0 e^{-x/p_0} + q_0 e^{-x/q_0} \]

Our aim is to find the extremal points of \( r(x) \). Hence,

\[ r'(x) = -p_0/x_p e^{-x/p_0} + q_0/x_q e^{-x/q_0} = 0 \]

The only point is

\[ x^* = x_p/x_q \left[ \log(x_q/x_p) + \log(p_0/q_0) \right]. \]

Note that \( r(0) = p_0/q_0 > 0 \), \( \lim_{x \to \infty} r(x) = +\infty \), \( r(x) < 0 \) for \( 0 < x < x^* \) and positive for \( x > x^* \): thus \( x^* \) is the unique point at which \( r(x) \) attains its minimum value and so \( r(x^*) \) is its global minimum.

Remark 1. The value \( x^* \) depends on the quantities \( p_0, q_0, x_p, x_q \). Not all choices make sense. For instance in [3] the authors proposed these (reasonable) choices.

\[ x_p = 1 \text{ and } x_q = 3. \]

The value for \( x_q \) comes from usual habit while \( x_p = 3 \) is related to the story of the dwarf Percheo and the hugest barrel in the world of the Heidelberg castle (see Fig. 1, Left). The inhabitants of the castle used to drink at least two liters of wine per day while the dwarf Percheo twelve bottles (about 6 liters). He was very healthy till the day he drank, for a lost bet, two glasses of water and suddenly he died maybe because the water, at those time, was polluted.

\[ p_0 \approx q_0. \]

This is a reasonable choice since both \( p_0 \) and \( q_0 \) represent the probability to get or to avoid any such disease by an abstainer.

For the choice, \( x_p = 1 \) and \( x_q = 3 \), \( p_0 = 0.2, q_0 = 0.25 \) we have the solution \( x^* \approx 0.76 \) liters (see Fig. 1, Right) which suggests that one bottle per day of red wine (possibly drunk during meals!)
is the suitable quantity that prevents from cardiovascular di- 
esses avoiding any liver disease. 

Apart the resveratrol, in the wine there are many other compoumds (at least 600 are very significant) and the drinking of a bottle of wine per day, as obtained above, if for someone can really prevent cardiovascular diseases for the majority of people this will lead to other physical problems. That is why this is a paradox. On the other hand, the properties of resveratrol are well studied and nowadays there are medicines based on the chemical components of resveratrol used to prevent the flu (one example of such medicines is ResVis).

The Mathematics of wine tasting

In this section we try to present various aspects of wine tasting technique, used by experts like sommeliers and enologists, that can be defined mathematically. In particular, when a wine is tasted, in order to understand its characteristics there are essentially three steps or analyses that must be done. The first one is the so-called visual analysis of the wine, in which one judges the color, the limpidity, the transparency and, in the case of sparkling wines, the consistency, i.e. the number and persistence of bubbles. A second check is the so-called olfactory examination where the experts should objectively make an analysis of the olfactory intensity, olfactory persistence and finding some correspondence with known scents and smells. The more scents one can clearly identify the more complex will be the wine. The third and essentially the last important aspect to check is the gustative analysis. Here one has to analyze the so-called soft and hard parts of a wine. The soft part is made up by the sugars, the alcohols and the poly-alcohols while the hard part, is made up by the acids, the tannins and the mineral salts. Finally, on the basis of the previous analysis one can say that a wine has a general structure which is thin up to vigorous and also say something about its harmony and evolution. For more details see, e.g., the reference [1]. It is not the aim of this paper to write a treatise on the techniques of wine tasting, but this small introduction can help us in understanding some of the considerations we are describing.

The geometry of the olfactory examination

In the olfactory examination one checks firstly two important features of the wine: the intensity and the complexity of the wine scents. The intensity represents the height of the scents while the complexity is their persisntency in time (i.e. its duration), that is their length. The sum of these two characteristics is the olfactory quality of our wine. Then, the more intense and complex is the wine, the better is its quality. Therefore, if we use a cartesian plane (Complexity, Intensity) the olfactory quality, \( q(x,y) \) is an increasing two-dimensional function. Obviously \( q(x,y) \) will not increase with the same gradient along \( x \) and \( y \) because, for instance, the intensity can be greater than the complexity. In Fig. 2 we show a possible mathematical representation of the quality of a wine as the monotone function \( q(x,y) = x + y^2 \). For this function we may say that the wine is more complex than intense.

Another aspect related to the olfactory complexity is the number and types of scents that one can identify in smelling the wine. These scents are usually grouped in three sets: primaries strictly connected to the particular kind of grapes and the environment, secondaries formed essentially during the wine fermentation and tertaries due to the evolution and ageing processes. The union of these three scents makes the wine olfactory complexity and the experts afterwards will say that a wine is {deficient, poorly complex, quite complex, complex, wide}.

Let \( O \) be the olfactory complexity. Thus, \( O \) is the function whose domain is the set

\[
D = P \cup S \cup T
\]

consisting of the union of the sets representing the primary, secondary and tertiary scents respectively, with values in the discrete set

\[
A = \{ \text{deficient, poorly complex, quite complex, complex, wide} \}.
\]

Remark 2. This functional way to describe the olfactory complexity, can be applied to almost all other aspects of wine tasting, such as in the description of the acidity or the alcohol (cf. [1]). In fact, sommeliers in describing a wine associate to any of the wine characteristics a scale of values. These scales represent indeed the images of the corresponding functions. [Segue al numero 114]

**ARTICOLI BREVI, MA TOSI**

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