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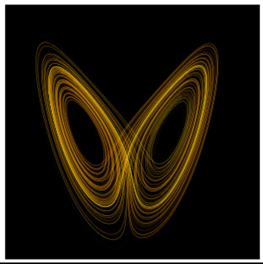

Some mathematics in the wine

Stefano De Marchi *

Part II [*Segue dal n.* 117]

They showed that when the barrels are new the diffusion kinetics measured in terms of the *rate of accumulation* of the compound, can be fit by exponentials functions of the rate, otherwise, when the barrels are already used, by polynomials of decreasing degree. In [9] the study of a *Fickian diffusion model* for simulating the wine losses during ageing in oak barrels, has been presented. This model is based on *Fick's second law* that models non-steady state diffusion processes, that is processes in which the concentration within the diffusion volume changes with respect to time. Letting $u(x,t)$ the concentration function (subject to some Dirichlet and Robin boundary conditions) the diffusion equation corresponding to Fick's second law is

$$u_t(x,t) = D\, u_{x,x}(x,t)$$

where D is the diffusion coefficient that depends on the type of wood. The interest of these two examples is the fact that the wine and the phenomena connected to the ageing is dynamically evolving as a diffusion process.

Wine as chaotic dynamical system

So far we have not yet discussed what is a dynamical system and in particular a chaotic dynamical system: this is what are going to say.
The ``system-wine'' depends on many variables, part of them come from the environment, especially soil and climate, and the others from the grapes, that combine in chaotic way. This is the intuitive reason why the wine can be considered a *chaotic dynamical system*. We will conclude (see next section) by suggesting a particular wine called ***Chaos***, a mix of montelpulciano, syrah and merlot. The wine is produced in the region Marche in Italy and its bottle's labels are different views of the Mandelbrot set.

To understand a dynamical system we need to know its *state*. The state of a dynamical system is determined by a collection of real numbers, or more generally by a set of points in an appropriate state space.

To be clearer, for modeling the dynamics of a system, we start from a set $x_1,\dots,x_n$ of measurable quantities that represent the system's state at the time t, that is

$$\mathbf{x}(t):=(x_1(t),\dots,x_n(t))$$

If t is a real number, the dynamical system is called *continuous* otherwise, when t is a natural number, the system is called *discrete*.

The evolution of system is formally a function, that once the initial state $\mathbf{x}_0$ at t_0 is known, allows to uniquely determine the state of the system in any successive time t:

$$\mathbf{x}_t=F(t_0,\mathbf{x}_0;t) \qquad (4)$$

Hence, starting from the initial condition $\mathbf{x}(t_0)$, the set of values obtained by (4) is the *trajectory* of the system passing through $\mathbf{x}_0(t_0)$.In practise, it is not easy to find the operator F and people try to recover it by using some differential equations representing the *local evolution* of the system. These equations, in the continuous setting, describe the variation of each state variable x_i w.r.t. the others and itself, too. That is

$$d\mathbf{x}_i/dt=f_i(x_1,\dots,x_n), \quad i=1,\dots,n.$$

Unfortunately only in simple cases, for example with linear differential equations, one can find the analytic solution of the system satisfying the initial conditions. Similar considerations can be done for *discrete* dynamical systems where difference equations will be considered instead of differential equations:

$$x_i(t+1)= f_i(x(t)).$$

It is beyond the aim of this paper to go further in the theory of dynamical systems. What we only want to understand is why wine is a *chaotic* dynamical system.

For a dynamical system to be classified as *chaotic*, most scientists will agree that it must have the following properties.

- It must be *sensitive* to initial conditions. That is an arbitrarily small perturbation of the current trajectory may lead to significantly different future behavior. Sensitivity to initial conditions is often confused with chaos in popular accounts.
- It must be *topologically mixing*, in the sense that the system will evolve over time so that any given region or open set of its phase space will eventually overlap with any other given region. Here, *mixing* is really meant to correspond to the standard intuition: the mixing of colored dyes or fluids is an example of a chaotic system.
- Its *periodic orbits* must be *dense*.

A ***simple example*** of a real chaotic system is the smoke of cigarettes. In fact, even if cigarettes are lighted in macroscopical similar conditions, their smoke can behave in very different way, depending on the air pressure, the air currents, the air temperature and so on.
Similarly, the fermentation of the must, that contains about 2000 known compounds, depends on the air pressure, humidity, temperature, the lunar phase, and so on. Therefore, small changes of these during the fermentation could influence significatively the production and also the evolution of the wine, so that we can get a *good wine* or a *"good vinegar"* Figure 4 shows these two chaotic dynamical systems: the match's smoke and the must.

Figure 4: (Right) A match's smoke and (Left) the fermenting must.

In many physical and mathematical models of wine fermentation kinetics, people mostly study the diffusion of some compounds and their effect on other compounds, such as yeast cells and sugars, or the diffusion of the flavors in oak barrels or due to oak chips. These models are essentially a description of chemical phenomena by means of biological mathematical models. As detailed in [11] the fermentation kinetics model can be subdivided into three parts: a growth mo-

del, a substrate model and a product model.
Recently the *sigmoidal logistic model* has been one of the most popular used for simulations due to its property of *good fit* of experimental data. In its standard form the sigmoidal curve is the solution of the Cauchy problem

$$dp/dt=p(1-p), \quad p(0)=p_0. \tag{5}$$

This first order non-linear differential equation (5) is indeed a special case of the well-known *Verhulst logistic model*

$$dp/dt=k\,p\,(1-p/C), \quad p(0)=p_0. \tag{6}$$

where the constant k represents the growth rate and C the carrying capacity of the system. This model, in the discrete case, is usually represented (after a scaling process) by the iterative map

$$x_{n+1}= \sigma x_n(1-x_n), \quad n \geq 0$$

which, for values of the parameter σ grater than 3.45, shows some chaotic behaviour that graphically produces the well-known *bifurcation diagram* (see Fig. 6, right).

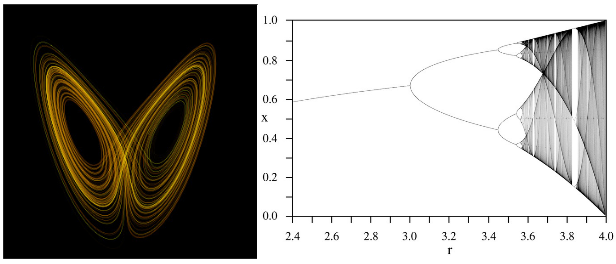

Figure 6: (left) Lorenz attractor, (right) the Logist map and its bifurcation diagram

Some dynamical systems are chaotic everywhere but in many cases chaotic behaviour is found only in a subset of the phase space. The cases of great interest arise when the chaotic behaviour takes place on an *attractor*, since then a large set of initial conditions will lead to orbits that converge to this chaotic region. While most of the motion types mentioned above give rise to very simple attractors, such as points and circle-like curves called *limit cycles*, chaotic motion gives rise to what are known as *strange attractors*, i.e. attractors that can have great detail and complexity. For instance, a simple three-dimensional model of the Lorenz weather system gives rise to the famous *Lorenz attractor* (see Figure 6 well-known for its butterfly shape. Strange attractors are also *fractals* whose most important representatives are the *Mandelbrot set* and the *Julia sets*, famous also as screen savers.
In conclusion, wine fermentation and wine ageing are not simple dynamical systems to study and the study of the wine fermentation should be better modeled by *reaction-diffusion equations* of the form

$$du(\mathbf{x},t)/dt=D\,\Delta\, u(\mathbf{x},t)+ K\, g(u(\mathbf{x},t)) \tag{7}$$

where the function g represents the reaction of the system, usually a non-linear function of *u*, with the function *u* depending on the vector $\mathbf{x}$, which represents the variables involved in the evolution, and the time t. For these equations the solution is usually found by sophisticated numerical methods (cf. e.g. [6]). Unfortunately the great number of substances that make up wine, make its behaviour often chaotic and therefore people solving numerically the reaction-diffusion equations of the wine evolution, should study other aspects connected to the numerical solution, such as stability.

"Chaos" is also a wine

We have just seen that wine is composed of at least 2000 compounds that combine and mix together and evolved in time, giving to the wine its "personality". An Italian wine producer, the firm "Fattoria Le Terrazze" whose vineyards are in Numana nearby Ancona, has decided since 1997 to call one of its red wines with the name ***Chaos***. The producer, on the basis of our previous discussion about the "chaos into the wine", decided to use as label of Chaos, views of the Mandelbrot set. In Figure 7 we show some labels of Chaos.

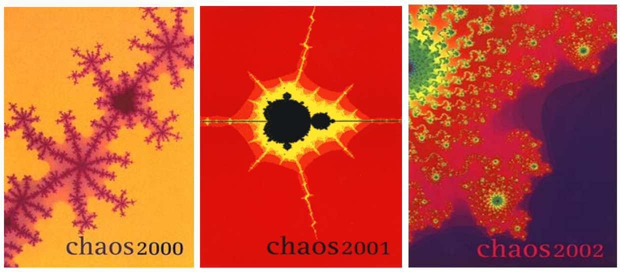

Figure 7: Chaos 2000, 2001 and 2002.

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Buchi neri, informazione, computazione a scale ultramicroscopiche

di Paolo Di Sia ([1,2]), Valerio Dallacasa ([1])

La fisica dei buchi neri

La fisica dei buchi neri iniziò a svilupparsi in maniera forte negli anni '60, in relazione anche a necessità riguardanti la ricerca di connessioni tra fisica quantistica e relatività generale. I concetti di ordine e disordine in fisica assumono un significato preciso in particolare in relazione al concetto più generale di correlazione tra livelli energetici. Maggiore è tale correlazione, maggiore è l'ordine del sistema, mentre ad una più larga distribuzione di livelli corrisponde un più alto valore dell'entropia. All'inizio di tali studi emerse che ogni informazione va completamente perduta all'interno di un buco nero a causa di quello che il fisico **J. A. Wheeler** ha denominato "**teorema NO-HAIR**" (dell'assenza di capelli) di un buco nero. Tale teorema restringe lo stadio finale di una stella collassante a sole 4 possibilità, che sono le 4 soluzioni delle equazioni di **Einstein** descriventi i buchi neri (abbreviati bn di seguito):

1) bn di Schwarzschild, di massa **M**, carica **Q=0**, **non rotanti**;
2) *bn di* Kerr*, di massa **M**, carica **Q=0**, **rotanti**;*
3) *bn di* Reisnerr-Nordstrom*, di massa **M**, carica **Q**, **non rotanti**;*
4) *bn di* Newmann*, di massa **M**, carica **Q**, **rotanti**.*

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