Introduction to Geometric Measure Theory

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Timetable: 24 hours. First lecture on March 12, 2018, 10:30 (dates already fixed, see the calendar), Torre Archimede Room 2BC/30.

Course requirements: Integration theory (with respect to arbitrary measures), basic notions of Functional Analysis

Examination and grading:

SSD: MAT/05

Aim:

Course contents:
The first part of the course is an introduction to some of the fundamental notions of Geometric Measure Theory having applications in several parts of Mathematical Analysis: Hausdorff measures and Hausdorff dimension, area formula, rectifiable sets. In the second part of the course a more specialized topic will be treated, namely, the theory of integral currents, which will be seen from the viewpoint of the Plateau problem (existence of minimal surfaces with prescribed boundary); this part will be preceded by a detailed presentation of the needed multilinear algebra notions.
Detailed program [topics between brackets might be skipped for time reasons]

- Review of the possible approaches to the Plateau problem
- Hausdorff measure and Hausdorff dimension
- [Hausdorff and capacitary dimensions, Frostman lemma]
- [Covering theorems and applicatios]
- [Self-similar fractals]
- Lipschitz functions, Rademacher theorem (via Sobolev spaces)
- Area formula
- [First variation of the area]
- [Coarea formula]
- Rectifiable and purely unrectifiable sets
- Tangent space to a rectifiable set

- Rectifiability criteria
- k-vectors and k-covectors, simple k-vectors and their geometric interpretation
- k-forms, orientation of submanifolds, Stokes theorem
- Definition of currents, boundary and mass
- Normal, rectifiable, integral and polyhedral currents
- Statement of Federer-Fleming compactness theorem
- Solution to the Plateau problem in the framework of integral currents
- [Basic regularity for minimal currents]
- Operations on currents: product, push-forward, cone on a given current
- Constancy lemma
- Flat norm and properties
- Polyhedral deformation theorem
- [Isoperimetric inequality]
- [Characterization of integral currents by slicing]
- [Boundary rectifiability theorem]
- [Proof of Federer-Fleming compactness theorem]