Optimal Stopping, Singular and Impulsive Stochastic Control and Applications in Economics and Finance

Giorgio Ferrari[1], Tiziano Vargiolu[2]

[1] *Center for Mathematical Economics, Univ. of Bielefeld, Germany*
Email: giorgio.ferrari@uni-bielefeld.de

[2] *Dipartimento di Matematica, Universitá di Padova*
Email: vargiolu@math.unipd.it

Timetable: 24 hrs., Part 1: October 2018 and Part 2: March 2019.
First lecture of Part 1 on October 24, 2018, 14:30 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Knowledge of stochastic analysis and ODE/PDE theory.

Examination and grading:

SSD: SEC-06

Introduction: In this course we will introduce a class of stochastic optimization problems in continuous time that finds relevant applications in Economics and Finance, but has nonethless independent importance for its mathematical beauty. These are optimal stopping problems, and singular and impulsive stochastic control problems. We will provide their formulation, and (some of) the methods of solutions through the study of important examples. In particular, we will concentrate on the so-called analytic approach, which brings back the stochastic problem to a suitable second-order partial differential problem, which in these particular stochastic control problems takes the form of so-called variational inequality (VI) or quasi-variational inequality (QVI), characterized by the fact that part of the problem is to find a so-called free boundary, which divides the domain into regions where different regimes are expected for the stochastic state process. These differential problems have the well-known peculiarity that their solution is typically non-smooth, in particular the second derivative is expected not to be continuous in all the domain, but has a discontinuity at the free boundary. We will also discuss the important link between optimal stopping, and singular and impulsive control problems. Finally, if time allows, we will present some examples where Volterra integral equations can be used to characterize the free boundary.

Course contents:

Optimal Stopping (ca. 8 hours):

- Formulation of optimal stopping problems.
- The Dynamic Programming Principle, the related Hamilton-Jacobi-Bellman equation (a VI), and a Verification Theorem.
- An example: optimal harvesting problem; the solution of the VI (and thus the value function) is not C^2.
- Pricing problem of perpetual American options in the Black-Scholes model;

- Discussion on the pricing of finite time-horizon put options in the Black-Scholes model: early exercise premium representation; (if time allows, an integral Volterra equation for the free boundary).

Singular Stochastic Optimal Control (ca. 8 hours):

- Formulation of the problem through a classical example;
- The related Hamilton-Jacobi-Bellman equation (a QVI), and a Verification Theorem.
- Connection to optimal stopping and discussion on methods of solutions.

Impulsive Stochastic Optimal Control (ca. 8 hours):

- Formulation of the problem through a classical example.
- The related Hamilton-Jacobi-Bellman equation (a QVI), and a Verification Theorem.
- An example: optimal repeated harvesting problem; the solution of the QVI (and thus the value function) is not C^2;
- Connection to iterated optimal stopping.

References

1. W.H. FLEMING, H.M. SONER, Controlled Markov Processes and Viscosity Solutions, 2nd edition. Springer (2006).
2. D. LAMBERTON, Optimal Stopping and American Options. Lecture Notes of the Ljubljana Summer School on Financial Mathematics, September 2009.
3. G. PESKIR, A. SHIRYAEV, Optimal Stopping and Free-Boundary Problems. Lectures in Mathematics ETH, Birkhauser (2006).
4. B. ØKSENDAL, A. SULEM-BIALOBRODA, Applied Stochastic Control of Jump Diffusions. Universitext, Springer (2007).
5. H. PHAM, Continuous-time Stochastic Control and Optimization with Financial Applications. Stochastic Modeling and Applied Probability 61, Springer 2008
6. N. TOUZI, Optimal Stochastic Control, Stochastic Target Problems, and Backward SDE. Fields Institute Monographs 29, Springer 2013.