introduction to Ergodic Theory

Prof. Benettin Giancarlo¹, Prof. Carlangelo Liverani²

¹Dipartimento di Matematica “Tullio Levi-Civita”, Università di Padova
Email: benettin@math.unipd.it
²Dipartimento di Matematica, Università Tor Vergata, Roma
Email: liverani@mat.uniroma2.it

Timetable: 16+8 hrs. First lecture on October 23, 2019, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements:

Examination and grading:

SSD: MAT/07

Aim:

Course contents:

Part I - G. Benettin (16 h)

Introduction; dynamical systems with measure; elementary examples; the Liouville measure for Hamiltonian systems; isomorphism and classification.
General results: the Birkhoff-Kinchin ergodic theorem; the Poincarè return theorem.
Excursus: the physical roots of ergodic theory; some basic ideas of Boltzmann and Gibbs.
The notion of ergodicity; examples. The notion of mixing; examples. The ergodic decomposition (hints).
The Kolmogorov-Sinai entropy: notion, main results, examples.
Possible additional topics, if there is time: the spectral approach to ergodic theory.

Part II - C. Liverani (8 h)

Abstract: Fluctuations around the average are of fundamental physical relevance (starting with the proof of the existence of atoms in Einstein’s 1905 seminal paper). Such fluctuations can appear in space averages (when many degrees of freedom are present) or in time averages (ergodic averages), or in both at the same time (hydrodynamics).

I will discuss the case of ergodic averages by analysing some simple non-trivial examples. This will allow to illustrate some surprising and fundamental differences between regular and chaotic motions. In addition, I will explain in which exact sense chaotic and random motions are similar.