Convex integration: from isometric embeddings to Euler and Navier Stokes equations

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Timetable: 16 hours, March/April 2020. Torre Archimede, Room 2BC/30.

Course requirements: Very basic notions of ODE and PDE theory and of differential geometry (Riemannian manifold, length of curves).

Examination and grading: Seminar

SSD: MAT/05 - Mathematical Analysis

Aim and Course contents:
Convex integration, first introduced by Nash to prove nonuniqueness of \(C^1\) isometric embeddings of Riemannian manifolds, turned out in the last ten years (starting from De Lellis and Székelyhidi) to be a very powerful tool to show nonuniqueness and flexibility of solutions (h-principle) in problems of fluid mechanics.

Aim of the course is to explain different applications of the technique of convex integration to some of these problems. In particular, we will focus on the solution of major problems like the Onsager’s conjecture on the existence of dissipative Hölder solutions to the Euler equations and the recent proof of nonuniqueness for weak (non Leray) solutions to the 3D Navier Stokes equations.

The first application of convex integration, namely that to the nonuniqueness of \(C^1\) isometric embeddings of Riemannian manifolds, will also be covered. The course should be particularly interesting for students in Mathematical Analysis, Differential Geometry and Mathematical Physics, in particular those interested in Fluid Mechanics.

References:
- L. Székelyhidi “From isometric embeddings to turbulence” Lecture Notes available online.
- S. Daneri, “Convex integration: from isometric embeddings to Euler and Navier stokes equations”, Lecture notes which will be given during the course.