Doctoral Program in Mathematical Sciences
Department of Mathematics “Tullio Levi-Civita”
University of Padova

Doctoral Program in Mathematical Sciences

Catalogue of the courses 2020

Updated October 1, 2019
INTRODUCTION

This Catalogue contains the list of courses offered to the Graduate Students in Mathematical Sciences for the year 2019-2020.

The courses in this Catalogue are of two types.

1. Courses offered by the Graduate School. This offer includes courses taught by internationally recognized external researchers. Since these courses might be not offered again in the near future, we emphasize the importance for all graduate students to follow them.

2. Some courses selected from those offered by the Graduate School in Information Engineering of the University of Padova, by the Master in Mathematics, and by other institutions, that we consider of potential interest for the students in Mathematics.

We underline the importance for all students to follow courses, with the goal of broadening their culture in Mathematics, as well as developing their knowledge in their own area of interest.

REQUIREMENTS FOR GRADUATE STUDENTS

Within the first two years of enrollment (a half of these requirements must be fulfilled within the first year) all students are required to follow and pass the exam of

- at least 2 among the courses called "Courses of the Doctoral Program" in this catalogue;
- other courses for a total commitment of at least 56 additional hours.

Students are warmly encouraged to take more courses than the minimum required by these rules, and to commit themselves to follow regularly these courses. At the end of each course the teacher will inform the Coordinator and the Secretary on the activities of the course and of the registered students.

Students must register to all courses of the Graduate School that they want to attend, independently of their intention to take the exam or not. We recommend to register as early as possible: the Graduate School may cancel a course if the number of registered students is too low. If necessary, the registration to a Course may be canceled.

Courses for Master of Science in “Mathematics”
Students have the possibility to attend some courses of the Master of Science in Mathematics and get credits for the mandatory 56 hours.
The recommendation of these courses must be made by the Supervisor and the amount of credits is decided by the Executive Board.

Courses attended in other Institutions
Students are allowed to take Ph.D. courses offered by PhD Programs of other Universities or in Summer Schools. Acquisition of credits will be subject to approval of the Executive Board.

Seminars
All students must attend the Colloquia of the Department and participate in the Graduate Seminar ("Seminario Dottorato"). They are also encouraged to attend the seminars of their research group.
HOW TO REGISTER AND UNREGISTER TO COURSES

The registration to a Course must be done online. Students can access the online registration form on the website of the Doctoral Course http://dottorato.math.unipd.it/ (select the link Courses Registration), or directly at the address http://dottorato.math.unipd.it/registration/.

In order to register, fill the registration form with all required data, and validate with the command “Register”. The system will send a confirmation email message to the address indicated in the registration form; please save this message, as it will be needed in case of cancellation.

Registration to a course implies the commitment to follow the course.

Requests of cancellation to a course must be submitted in a timely manner, and at least one month before the course (except for courses that begin in October and November) using the link indicated in the confirmation email message.

REQUIREMENTS FOR PARTICIPANTS NOT ENROLLED IN THE GRADUATE SCHOOL OF MATHEMATICS

The courses in this catalogue, although part of activities of the Graduate School in Mathematics, are open to all students, graduate students, researchers of this and other Universities.

For reasons of organization, external participants are required to communicate their intention (loretta.dallacosta@unipd.it) to take a course at least two months before its starting date if the course is scheduled in January 2020 or later, and as soon as possible for courses that take place until December 2019.

In order to register, follow the procedure described in the preceding paragraph. Possible cancellation to courses must also be notified.
Courses of the Doctoral Program

1. Prof. Giancarlo Benettin, Prof. Carlangelo Liverani
   Introduction to Ergodic Theory  
   DP-1

2. Prof. Bruno Chiarellotto, Prof. Matteo Longo
   The Arithmetic of Elliptic Curves  
   DP-2

3. Prof. Francesco Esposito
   Introduction to Quantum Groups  
   DP-4

4. Prof. Markus Fischer
   Stochastic Differential Equations and Applications  
   DP-5

5. Prof. Luca Martinazzi
   Degree theory and applications to geometry and analysis  
   DP-7

Courses of the “Computational Mathematics” area

1. Prof. Immanuel Bomze
   Introduction to Conic Optimization  
   MC-1

2. Prof. Alessandra Buratto
   Introduction to differential games  
   MC-2

3. Prof. Antoine Jacquier
   A smooth tour around rough models in finance. (From data to stochastics to machine learning) 
   MC-3

4. Prof. Sergei Levendorskii
   Fourier-Laplace Transform and Wiener-Hopf Factorization in Finance, Economics and Insurance  
   MC-4

5. Prof. Gianmarco Manzini
   Introduction to the Virtual Element Method and to numerical methods for PDEs on unstructured polytopal meshes  
   MC-6

6. Dott.ssa Elena Sartori
   Modeling interacting agents in social sciences  
   MC-7

7. Dr. Francesco Tudisco
   Eigenvector methods for learning from data on networks  
   MC-9

8. Prof. Tiziano Vargiolu
   Topics in Stochastic Analysis  
   MC-10
Courses of the “Mathematics” area

1. Prof. Andrei Agrachev, Prof. Davide Barilari
   Introduction to Subriemannian geometry M-1

2. Prof. Fabio Ancona, Prof. Massimiliano D. Rosini
   Introduction to Hyperbolic Conservation Laws M-2

3. Prof. Sara Daneri
   Convex integration: from isometric embeddings to Euler and Navier Stokes equations M-3

4. Prof. Tom Graber
   Intersection Theory in Algebraic Geometry M-4

5. Prof. Alexey Karapetyants
   Morrey-Campanato Spaces and classical operators M-5

6. Prof. Elena Mantovan
   Introduction of Shimura Varieties M-7

7. Prof. Marco Mazzucchelli
   Introduction to Floer Homology M-8

8. Prof. Vitaly Moroz
   Positivity principles and decay of solutions in semilinear elliptic problems. M-10

9. Prof. Ivan Penkov
   Topics in the representation theory of infinite-dimensional Lie algebras M-12

10. Prof. Leonid Positselki
    Contramodules in tilting theory and applications to the Enochs conjecture M-14

11. Prof. Franco Rampazzo
    Introduction to Optimal Control Theory M-15

12. Prof. Andrea Santi
    An introduction to Supergravity in 11-dimensions M-16

Courses offered within the Masters’s Degree in Mathematics

1. Offered Courses MD-1

Soft Skills
Courses of the Doctoral Program
introduction to Ergodic Theory

Prof. Benettin Giancarlo¹, Prof. Carlangelo Liverani²

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²Dipartimento di Matematica, Università Tor Vergata, Roma
Email: liverani@mat.uniroma2.it

Timetable: 16+8 hrs. First lecture on October 23, 2019, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements:

Examination and grading:

SSD: MAT/07

Aim:

Course contents:

Part I - G. Benettin (16 h)

Introduction; dynamical systems with measure; elementary examples; the Liouville measure for Hamiltonian systems; isomorphism and classification.
General results: the Birkhoff-Kinchin ergodic theorem; the Poincarè return theorem.
Excursus: the physical roots of ergodic theory; some basic ideas of Boltzmann and Gibbs.
The notion of ergodicity; examples. The notion of mixing; examples. The ergodic decomposition (hints).
The Kolmogorov-Sinai entropy: notion, main results, examples.
Possible additional topics, if there is time: the spectral approach to ergodic theory.

Part II - C. Liverani (8 h)

Abstract: Fluctuations around the average are of fundamental physical relevance (starting with the proof of the existence of atoms in Einstein’s 1905 seminal paper). Such fluctuations can appear in space averages (when many degrees of freedom are present) or in time averages (ergodic averages), or in both at the same time (hydrodynamics).

I will discuss the case of ergodic averages by analysing some simple non-trivial examples. This will allow to illustrate some surprising and fundamental differences between regular and chaotic motions. In addition, I will explain in which exact sense chaotic and random motions are similar.
The Arithmetic of Elliptic Curves

Prof. Bruno Chiarellotto¹, Prof. Matteo Longo²

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²Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova
Email: mlongo@math.unipd.it

Timetable: 24 hrs. 2020, Room 2BC/30, Torre Archimede

Course requirements:

Examination and grading:

SSD: MAT/02-03

Aim:
I propose to give an introduction to some of the previous topics and, if time permits, to explain what are some of the known results on this conjecture. Each of the topic can be very technical (especially Wiles’s proof). Instead of explaining complete proofs, which would be clearly out of the scope of this course, I would like to propose an overview of the topics and the main results, giving some hint, when possible, on the techniques which are used to attack the problems.

The plan of the lectures (24h) is as follows:

1. Introduction to the arithmetic theory of elliptic curves (6h).
2. The Mordell-Weil Theorem (2h).
3. The L-function of an elliptic curve (2h)
4. Wiles Modularity Theorem, modular forms and Fermat Last Theorem (6h).
5. The Birch and Swinnerton-Dyer Conjecture (BSD) (4h).
6. Results on the BSD Conjecture: Heegner points and the work of Kolyvagin (4h).

Course contents:
Number Theory is a wide branch of pure mathematics which studies a number of different problems having to do (in a vague sense) with arithmetic proprieties of numbers such as the field of rational numbers. For example, one of the main fascinating problems in number theory is to describe the structure of the Galois group Gal(\overline{\mathbb{Q}}/\mathbb{Q}) of the algebraic closure of \mathbb{Q}. It is quite common in number theory to study a problem by means of a great number of different techniques, going from the complex analysis to representation theory, from dynamic systems to algebraic geometry, from functional analysis to abstract algebra. In the previous example, if one looks at representations \rho : Gal(\overline{\mathbb{Q}}/\mathbb{Q}) \rightarrow GL_2(\mathbb{C}), one finds in a natural way that an interesting subset of these is made out of complex analytic functions (called modular forms) having a large number of symmetries, which can be studies by complex analytic method on one side, and by means of techniques from algebraic geometry on the other side (modular forms also appear in other field of mathematics, and in recent years also in physics). It is the mix of these techniques...
which makes the subject difficult and, at the same time, fascinating. Let us remark that two of the Millennium Problems are about Number Theory: The Riemann Hypothesis and the Birch and Swinnerton-Dyer conjecture. The purpose of this course is to give a brief introduction to one of the two Millennium Problems alluded to before: the Birch and Swinnerton-Dyer conjecture. To explain the problem, one considers an elliptic curve given by an affine equation

\[ E : y^2 = f(x) \]

with \( f(x) \) a non-singular cubic polynomial with coefficients in \( \mathbb{Z} \). When we add the point at infinity (i.e. we consider this curve in the projective plane) we obtain a projective non-singular curve, whose points \( E(F) \) over any finite field extension \( F \) of the field of rational numbers can be equipped with a structure of finitely generated abelian group (Mordell-Weil Theorem). In particular, we can write

\[ E(\mathbb{Q}) \cong \mathbb{X}^r \oplus E(\mathbb{Q})_{\text{tors}} \]

where \( E(\mathbb{Q})_{\text{tors}} \) is a finite group. The integer \( r \geq 0 \) appearing above is the algebraic rank of \( E \). On the other hand, one can construct a function \( s \mapsto L(E/\mathbb{Q}, s) \) of the complex variable \( s \) as a convergent (for the real part of \( s \) greater than 3/2) product of factors, one for each prime number \( p \), such that each of these factors counts the number of points (with coordinates in the field with \( p \) elements) of the curve obtained by reducing the coefficients of \( f(x) \) modulo \( p \) (note the similarity with similar products for the Riemann Zeta function). Thanks to the work of Wiles (leading the the proof of Fermat Last Theorem) one knows that this complex function can be extended to all \( \mathbb{C} \) to an entire function, having a functional equation with center of symmetry \( s = 1 \). The Birch and Swinnerton-Dyer Conjecture states that the algebraic rank of \( E(\mathbb{Q}) \) is equal to its analytic rank, namely, the order of vanishing of \( L(E/\mathbb{Q}; s) \) at \( s = 1 \). (One also has a more precise description of the leading value in terms of fine arithmetic invariants of \( E \).) This is a deep conjecture because relates object of completely different nature, algebraic and analytic.
Introduction to Quantum Groups

Prof. Francesco Esposito¹

¹Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova
Email: esposito@math.unipd.it

Timetable: 24 hrs., Apr/May 2020, Torre Archimede, Room 2BC/30.

Course requirements:

Examination and grading:

SSD: MAT/02-03

Aim:

Quantum groups arose at first in connection with problems in statistical mechanics and are closely related to conformal field theory. Moreover, they have applications to many different areas of mathematics, e.g. knot theory and topology, the study of the absolute Galois group of \( \mathbb{Q} \), representation theory of algebraic groups in characteristic \( p \), Poisson-Lie groups, the theory of \( (q) \)- special functions. They have also served as a rich source of examples in non-commutative geometry. For these reasons, the theory of quantum groups may be of interest to mathematicians with expertise in any of the following: algebra, number theory, geometry, topology, mathematical physics, analysis. The intent of this course is to offer an introduction to quantum groups and survey its diverse applications. Being aimed at a general audience, its prerequisites are limited to the basics of linear algebra and elementary notions of topology. Any other needed notion will be introduced through examples or formal definition.

Course contents: (Tentative)

1. History and Motivations, applications. Basic notions of representation theory.
2. The Lie algebra \( \mathfrak{sl}_2 \) and its representations (characteristic zero and modular). The universal enveloping algebra \( U(\mathfrak{sl}_2) \) of \( \mathfrak{sl}_2 \).
3. The quantized enveloping algebra \( U_q(\mathfrak{sl}_2) \) for \( q \) generic and \( q \) a root of unity.
5. Quantized universal enveloping algebras. PBW theorem. Specializations and its center
8. Quasi-Hopf algebras and applications to the absolute Galois group of \( \mathbb{Q} \).
10. Representations of Quantized universal enveloping algebras for generic \( q \).
Stochastic Differential Equations and Applications

Prof. Markus Fischer\(^1\)

\(^1\)Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova
Email:fischer@math.unipd.it

**Timetable:** 24 hrs. May/June 2020, Torre Archimede, Room 2BC/30.

**Course requirements:**
Basics of real analysis and measure-theoretic probability. Familiarity with continuous-time stochastic processes is advantageous, but not required.

**Examination and grading:**
SSD: MAT/06

**Aim:**
Introduce fundamental results and techniques in the theory of stochastic differential equations (SDEs) and illustrate their connections with ordinary and certain classes of partial differential equations. Some applications to optimal control, many-particle systems, and their mutual interplay will be treated.

**Scope:** Focus on SDEs driven by finite-dimensional Wiener processes.

**Course contents:**
1. Itô calculus and variants of Itô’s formula. SDEs driven by Brownian motion (Wiener processes). Questions of existence and uniqueness in weak and strong solutions, results by Yamada-Watanabe. Strong solutions for non-globally Lipschitz coefficients, weak solutions through the Girsanov transformation.
3. Kolmogorov backward and Kolmogorov (Fokker-Planck) forward equation. Differentiation of SDEs with respect to initial conditions and probabilistic solution of linear parabolic PDEs. Feynman-Kac and Bismut-Elworthy-Li formulae. Regularization by noise for ODEs. 1 / 2
4. Stochastic optimal control, dynamic programming and Hamilton-Jacobi- Bellman equation. Control through change-of-measure and backward SDEs. Variational representation of relative entropy and large deviations from the small noise limit.
5. McKean-Vlasov SDEs, nonlinear Kolmogorov equation, and propagation of chaos for mean field systems.
6. If time permits and depending on participants’ interests, introduction to optimal control of McKean-Vlasov SDEs or to mean field games.
References:


Degree theory and applications to geometry and analysis

Prof. Luca Martinazzi

1 Dipartimento di Matematica “Tullio Levi-Civita”, Università di Padova
Email: martinaz@math.unipd.it

Timetable: 24 hrs. First lecture on October 7, 2019, 14:00 (dates already fixed see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Analysis and basic differential geometry.

Examination and grading: oral exam.

SSD: MAT/05, MAT/03

Aim: Give the basic notions and some beautiful applications of degree theory in analysis and geometry.

Course contents:
The course starts with an elementary introduction to submanifolds in $\mathbb{R}^n$ and to the differentiable functions among them, in order to present the notion of Brouwer degree of a function and its fundamental properties (particularly the homotopy invariance). With elementary but non-trivial methods we use the Brouwer degree to prove:

- The fundamental theorem of algebra
- The fixed-point theorem of Brouwer
- The sphere theorem (there are no non-vanishing tangent vector fields on an even-dimensional sphere)
- The separation theorem of Jordan in $\mathbb{R}^n$.

We then introduce the equivalent definition of de Rahm degree in terms of differential forms, used to prove a first version of the theorem of Gauss-Bonnet. Later we will work on tangent vector fields of a submanifold of $\mathbb{R}^n$, introducing the notion of index of a vector field and proving the famous theorem of Poincaré-Hopf. This uses Morse theory, which we will also discuss in a self-contained way. Using the Poincaré-Hopf theorem we then give a complete proof of the Gauss-Bonnet theorem for submanifolds of $\mathbb{R}^n$ of arbitrary dimension compact and without boundary. This is an elegant proof, normally not seen in standard courses of differential geometry.

We also discuss the relation between the Brouwer degree in $\mathbb{R}^n$, the winding number and the Cauchy formula from complex analysis.

In the last part of the course we develop the notion of Leray-Schauder degree and obtain the Caccioppoli-Schauder fixed-point theorem in Banach spaces, an infinite-dimensional analog of the fixed-point theorem of Brouwer. This will be used to prove Peano’s theorem on the existence of solutions to ODEs with continuous field. We will also use the Leray-Schauder degree to prove the existence of solutions to some nonlinear elliptic equations.
Bibliography:

2. J. Milnor, Topology from the differentiable viewpoint, Princeton 1965
Courses of the “Computational Mathematics” area
Conic, especially copositive optimization

Prof. Immanuel Bomze\textsuperscript{1}

\textsuperscript{1}Dept. Applied Mathematics and Statistics, University of Vienna
Email: immanuel.bomze@univie.ac.at

**Timetable:** 8 hrs, First lecture April 22, 2020 (tentative), Torre Archimede, Room 2BC/30.

**Course requirements:**

**Examination and grading:**

**SSD:** MAT/09

**Aim:**

**Course contents:**

Quite many combinatorial and some important non-convex continuous optimization problems admit a conic representation, where the complexity of solving non-convex programs is shifted towards the complexity of sheer feasibility (i.e., membership of the cone which is assumed to be a proper convex one), while structural constraints and the objective are all linear. The resulting problem is therefore a convex one, and still equivalent to some NP-hard problems with inefficient local solutions despite the fact that in the conic formulation, all local solutions are global.

Using characterizations of copositivity, one arrives at various approximations. However, not all of these are tractable with current technology. In this course, we will address some approaches on which tractable SDP- or LP-approximations, and also branch-and-bound schemes, may be based.
Introduction to differential games

Prof. Alessandra Buratto

\textsuperscript{1} Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova
Email: buratto@math.unipd.it


Course requirements: Basic notions of Differential equations and Optimal control

Examination and grading: Homework assignments and final test

SSD: SECS-S/06

Aim: Differential games are very much motivated by applications where different agents interact exhibiting an inter-temporal aspect. Applications of differential games have proven to be a suitable methodology to study the behaviour of players (decision-makers) and to predict the outcome of such situations in many areas including engineering, economics, military, management science, biology and political science. This course aims to provide the students with some basic concepts and results in the theory of differential games, as well as some applications in Economics and Management Science

Course contents:

Part 1: General theory
  - Recall of basic concepts of game theory, equilibrium (Nash ...)
  - Dynamic games: formalization of a differential game
  - Simultaneous and competitive differential games (Nash Equilibrium)
  - Hierarchic differential games (Stackelberg equilibrium)
  - Time consistency and perfectness
  - Differential games with special structures (linear quadratic games, linear state games)

Part 2: Applications in Economics and Management Science
  - Advertising and Promotion in a Marketing Channel.
  - Consignment contracts with cooperative programs and price discount mechanisms in a dynamic supply chain
  - Competition between national brands and private labels in a vertical channel structure

References:
A smooth tour around rough models in finance
(From data to stochastics to machine learning)

Prof. Antoine Jacquier

Imperial College, Londra
Email: a.jacquier@imperial.ac.uk

Timetable: 16 hrs, end March/beginning April 2020, Torre Archimede, Room 2BC/30.

Course requirements: Probability and Stochastic Calculus

Examination and grading: oral examination on the topics covered during the course

SSD: MAT/06, SECS-S/06

Aim: Aim: the course aims at introducing the recent theory on rough volatility models, namely stochastic volatility models in finance driven by the fractional Brownian motion. This class of models will naturally arise by looking at market data and at the end of the course the PhD student will have full control of advanced tools in stochastic calculus which are crucial in modern finance.

Course contents:

A quick glance at time series in market data (Equities, Currencies, Commodities, Rates...) leaves no doubt that volatility is not deterministic over time, but stochastic. However, the classical Markovian setup, upon which a whole area of mathematical finance was built, was recently torn apart when Gatheral-Jaisson-Rosenbaum showed that the instantaneous volatility is not so well behaved and instead features memory and more erratic path behaviour. Rough volatility was born. This new paradigm does not come for free, though, and new tools and further analyses are needed in order to put forward the benefits of this new approach. The goal of this course is to explain how Rough Volatility naturally comes out of the data, and to study the new techniques required to use it as a tool for financial modelling. We shall endeavour to strike a balance between theoretical tools and practical examples, and between existing results and open problems. The contents shall span, with more or less emphasis on each topic, the following:

2. Constructing a model consistent between the historical and the pricing measure: joint calibration of SPX and VIX options.
3. Pricing options in rough volatility models: from Hybrid Monte Carlo to Deep learning

The first item is anchored in fairly classical Statistics and Probability, while the second deals with Stochastic analysis. The last item draws upon recent literature connecting Path-dependent PDEs, Backward SDEs and Deep Learning technology. Prior knowledge in all areas is not required, but good Probability/Stochastic analysis background is essential.
Fourier-Laplace transform and Wiener-Hopf factorization in Finance, Economics and Insurance

Prof. Sergei Levendorskii

1Calico Science Consulting, Austin, TX
Email:

Timetable: 12 hrs. First lecture on June 22, 2020, Torre Archimede, Room 2BC/30.
Course requirements: Probability and Stochastic Calculus
Examination and grading: oral examination on the topics covered during the course
SSD: Mat/06, SECS-S/06

Aim: The course aims at introducing recent fast and robust pricing techniques for exotic derivatives (such as Bermudan and American) in general Lévy models.

Course contents:

The Fourier-Laplace transform and Wiener-Hopf factorization are ubiquitous in Mathematics, Physics, Engineering, Probability, Statistics, Insurance and Finance. Recently, several difficult problems in Game Theory and Economics were solved using the Wiener-Hopf factorization techniques. From the analytical viewpoint, problems considered in the course can be reduced to a sequence of calculations, each involving either the Fourier (or inverse Fourier) transform of a given function, or the convolution of two given functions. In turn, each of these operations can be performed numerically with high efficiency using the standard fast Fourier and Hilbert transforms and fast convolution. We introduce new more efficient versions of the fast Fourier and Hilbert transforms. The second general topic of the course is the new general methodology for efficient evaluation of integrals with integrands analytic in regions around the lines of integration, examples being numerical Fourier-Laplace inversion, calculation of the Wiener-Hopf factors and high transcendental functions. We introduce three families of conformal deformations of the contour of integration in the Fourier inversion formula and the corresponding changes of variables, which lead to much faster and more accurate calculations. The third general topic of the course is the EPV (expected present value operators) method. The strength of the EPV method stems from the interaction of the probabilistic and analytical techniques. In the standard analytical approach to solution of boundary problems, the operators are interpreted as the expectation operators. This allows one to relatively easily evaluate complicated expectations and solve optimal stopping problems with non-standard payoffs. All topics in the course and additional topics will be covered in S. Boyarchenko, M. Boyarchenko, N. Boyarchenko, and S. Levendorskij. Spectral Methods in Finance, Economics and Insurance. Springer, New York, 2020, the monograph in preparation for Springer due to be finished in this Fall. The full lists of references for the lectures and a more detailed contents’ list will be given during the first lecture.

- Lecture 1. Lévy models
• Lecture 2. Evaluation of probability distributions and pricing European options in Lévy models
• Lecture 3. Simplified trapezoid rule, Fast Fourier Transform and its variations
• Lecture 4. Conformal acceleration techniques
• Lecture 5. Barrier options with discrete monitoring and Bermudan options. Calculations in the state space
• Lecture 6. Barrier options with discrete monitoring and Bermudan options. Calculations in the dual space
• Lecture 7. Wiener-Hopf factorization
• Lecture 8. Contingent claims with continuous monitoring, boundary value problems and Wiener-Hopf factorization
• Lecture 9. Options with continuous monitoring, cont-d
• Lecture 10. Affine models
• Lecture 11. American options with infinite time horizon
• Lecture 12. American options with finite time horizon
Introduction to the Virtual Element Method and to numerical methods for PDEs on unstructured polytopal meshes

Prof. Gianmarco Manzini

1CNR-IMATI, Pavia
Email: marco.manzini@imati.cnr.it

Timetable: 16 hrs. First lecture on January 21, 2020, 09:00 (dates already fixed see calendar), Torre Archimede, Room 2BC/30.

Course requirements: basic notions of numerical analysis and numerical methods for partial differential equations (Finite Elements, Finite Volumes, Finite Differences).

Examination and grading: brief presentation on a course-pertinent subject and oral examination on the topics covered during the course.

SSD: MAT/08 - Numerical Analysis

Aim: The course aims at introducing the fundamental ideas and results on numerical methods for solving partial differential equations of elliptic and parabolic type, with special emphasis on the Virtual Element method.

Course contents:

- Week 1, Lecture 1: introduction to numerical methods for partial differential equations of elliptic types on unstructured meshes:
  - Polygonal Finite Element method (PFEM);
  - Mimetic Finite Difference (MFD) method;
  - Virtual Element method (VEM);
  - other variants (wG, HHO, HDG, etc).

- Week 1, Lecture 2: the conforming VEM; construction of the basic method, convergence analysis and implementation

- Week 2, Lecture 3: the nonconforming formulation; construction of the basic method, convergence analysis and implementation

- Week 2, Lectures 4: enhanced and serendipity formulations of the virtual element method

- Week 3, Lectures 5 and 6: the mixed formulation of the virtual element method

- Week 4, Lectures 7 and 8: virtual de Rham sequences and applications to electromagnetism and Stokes
Modeling interacting agents in social sciences

Dott.ssa Elena Sartori

1Dipartimento di Matematica “Tullio Levi-Civita”, Università di Padova
Email: esartori@math.unipd.it


Course requirements: Basic knowledge of probability theory.

Examination and grading: Oral exam

SSD: MAT06; SECS-S06

Aim: In this course we consider a class of stochastic models describing the collective behavior of a large system of interacting agents, where the interaction is of the mean-field type and the state space is finite. We analyze some of the main approaches proposed during the last 20 years in literature, paying particular attention to the techniques used for the case. From an applicative viewpoint modeling microscopic interaction between individuals allows to describe macroscopic phenomena such as, for instance, polarization of ideas or adoption rates of a new technology seen as the result of conformism, imitation or peer pressure. In the last decades in the context of social sciences, the first proposed models were inspired by statistical mechanics; in that setting each agent could update his state at most one per time following some probabilistic rates of transition with sequential updating, thus defining a Markov chain. The introduction of a random utility function defining agents rationality led to models closer to reality. What these models do not account for is strategic interaction: when deciding, agents forecast the action of others. This is crucial since the single agent payoff, in fact, depends explicitly on some function of the actions of others. This new approach leads to move to a game-theoretic framework, where the emerging dynamics is a controlled Markov process.

Course contents:

1. overview of models inspired by statistical mechanics describing the macroscopic limit of a large system of interacting agents;
2. introduction of the so called Random Utility Model and its time evolution, focusing on some techniques used to analytically tackle it, e.g. weak convergence of stochastic processes (convergence of generators);
3. basic tools of game theory (normal form games, Nash equilibrium, best response map, ...);
4. basic notions in control theory (optimal control problems, dynamic programming, HJB equations, ...);
5. an example of mean field game for spin systems;
6. formalization of a finite-state mean-field game.
References:

Eigenvector methods for learning from data on networks

Dr. Francesco Tudisco

1GSSI – Gran Sasso Science Institute – L’Aquila (AQ)
Web: http://personal.strath.ac.uk/f.tudisco/
Email: francesco.tudisco@gssi.it

Timetable: 12 hrs. February 2020 (date to be scheduled), Torre Archimede, room 2BC/30.

Course requirements: Standard background in numerical linear algebra, numerical and mathematical analysis.

Examination and grading: Oral presentation or written essay

SSD: MAT/08 Numerical Analysis; INF/01 ComputerScience; MAT/05 Mathematical Analysis

Aim: Provide an introduction to spectral methods for unsupervised and semisupervised learning tasks and centrality on networks based on eigenvectors of matrices, tensors, and multihomogeneous mappings.

Course contents: Graphs are fundamental tools for learning from data and for the analysis of complex systems. In fact, we often state questions and develop analysis in terms of nodes and edges. For example, we can describe a social system by modeling individuals as nodes and social interactions as edges between pair of nodes. Similarly, we can model a biochemical reaction by assigning a node to each protein and edges between them to model physical contacts of high specificity, so-called protein-protein interactions.

Due to the broad scope of this modeling paradigm, the analysis of systems or datasets as networks has enjoyed a tremendous success over the last decade and many mathematical models and numerical methods for handling learning problems on networks have been developed, based on eigenvectors and singular vectors of matrix, tensors (or hypermatrices) and, more generally, multihomogeneous mappings.

The course will give an introduction to spectral methods for learning from network data from a numerical linear algebra perspective by discussing the following topics in various detail:

- Graph Laplacian, spectral partitioning, semisupervised learning
- $p$-Laplacian and nonlinear spectral clustering
- Lovász extension and homogeneous mappings
- Eigenvector centrality and centrality based on matrix functions
- Centrality in higher order networks (time-varying, multilayer, hypergraphs)
- Nonlinear Perron–Frobenius theory and nonlinear power methods

References:

Topics in Stochastic Analysis

Prof. Tiziano Vargiolu

1Università di Padova
Dipartimento di Matematica “Tullio Levi-Civita”
Email: vargiolu@math.unipd.it

Calendario: 8 hrs. First lecture on October 9, 2019, 14:30 (dates already fixed, see the calendar), Torre Archimede, Room 2BC/30.
A preliminary meeting with the Students will be held on October 7, 2019, 10:00, Torre Archimede, Room 2BC/30.

Prerequisiti: A previous knowledge of the basics of continuous time stochastic analysis with standard Brownian motion, i.e. stochastic integrals, Itô formula and stochastic differential equations, as given for example in the master course ”Analisi Stocastica”.

Tipologia di esame: Seminar

SSD: MAT/06

Programma: The program will be fixed with the audience according to its interests. Some examples could be:

- continuous time stochastic control;
- Levy processes;
- numerical methods;
- stochastic control.
Courses of the “Mathematics” area
Introduction to Subriemannian geometry

Prof. Andrei Agrachev\textsuperscript{1}, Prof. Davide Barilari\textsuperscript{2}

\textsuperscript{1}Scuola Internazionale Superiore di Studi Avanzati (SISSA), Trieste
Email: agrachev@sissa.it
\textsuperscript{2}Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova
Email: barilari@math.unipd.it

**Timetable:** 16–20 hrs. April 2020, Torre Archimede, Room 2BC/30.

**Course requirements:**

**Examination and grading:**

**SSD:** MAT/05

**Aim:**

**Course contents:**
Introduction to Hyperbolic Conservation Laws

Prof. Fabio Ancona¹, Prof. Massimiliano Daniele Rosini²

¹Dipartimento di Matematica, Università di Padova
Email: ancona@math.unipd.it
²Dipartimento di Matematica e Informatica, Università di Ferrara
Email: massimilianodaniele.rosini@unife.it

Timetable: 16 hrs., on April/May 2020, Torre Archimede, Room 2BC/30.

Course requirements: very basic notions of ODE and PDE theory

Examination and grading: seminar

SSD: MAT/05 - Mathematical Analysis

Aim: the course aims at providing an introduction to:
- fundamental features of the theory of hyperbolic conservation laws in one space variable;
- topics in recent research on traffic flow models and networks for this class of first order non-linear PDEs.

The course shall be of particular interest for students in Mathematical Analysis, Mathematical Physics, Numerical Analysis, especially if interested in fluid dynamics models.

Course contents:
Part 1

Part 2
Conservation laws with discontinuous flux and with point constraints. Analysis of traffic flow models via vanishing viscosity and many particle approximations (micro-macro limit).

References:

• C.M.Dafermos, Hyperbolic Conservation Laws in Continuum Physics, Fourth, ed. Springer Verlag.
• M. Garavello, K. Han, and B. Piccoli, Models for vehicular traffic on networks, AIMS, 2016.
Convex integration: from isometric embeddings to Euler and Navier Stokes equations

Sara Daneri

1Gran Sasso Science Institute, L’Aquila, Italy
Email: sara.daneri@gssi.it

Timetable: 16 hours, March/April 2020. Torre Archimede, Room 2BC/30.

Course requirements: Very basic notions of ODE and PDE theory and of differential geometry (Riemannian manifold, length of curves).

Examination and grading: Seminar

SSD: MAT/05 - Mathematical Analysis

Aim and Course contents:
Convex integration, first introduced by Nash to prove nonuniqueness of $C^1$ isometric embeddings of Riemannian manifolds, turned out in the last ten years (starting from De Lellis and Székelyhidi) to be a very powerful tool to show nonuniqueness and flexibility of solutions (h-principle) in problems of fluid mechanics.

Aim of the course is to explain different applications of the technique of convex integration to some of these problems. In particular, we will focus on the solution of major problems like the Onsager’s conjecture on the existence of dissipative Hölder solutions to the Euler equations and the recent proof of nonuniqueness for weak (non Leray) solutions to the 3D Navier Stokes equations.

The first application of convex integration, namely that to the nonuniqueness of $C^1$ isometric embeddings of Riemannian manifolds, will also be covered. The course should be particularly interesting for students in Mathematical Analysis, Differential Geometry and Mathematical Physics, in particular those interested in Fluid Mechanics.

References:
- L. Székelyhidi “From isometric embeddings to turbulence” Lecture Notes available online.
- S. Daneri, “Convex integration: from isometric embeddings to Euler and Navier stokes equations”, Lecture notes which will be given during the course.
Intersection Theory in Algebraic Geometry

Tom Graber

The Division of Physics, Mathematics and Astronomy, California Institute of Technology
Email: graber@caltech.edu

Timetable: 16 hrs, First lecture on October 8, 2019, 09:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Students should have basic knowledge of algebraic geometry and commutative algebra. It will also be helpful (although not logically necessary) to be familiar with properties of topological homology and cohomology of manifolds.

Examination and grading:

SSD: MAT/03

Aim: The goal is to develop basic concepts and techniques of intersection theory and to see how they can be applied to problems in algebraic geometry.

Course contents: Some topics that we will definitely cover will be algebraic cycles, rational equivalence, Chern classes, and intersection products. Depending on student interests, additional topics could include the Grothendieck–Riemann–Roch Theorem, Chow rings of homogeneous spaces and relation to enumerative geometry, equivariant Chow groups, or others.

References:

Good references for the course would be Intersection Theory by Fulton and/or 3264 and All That by Harris and Eisenbud.
Morrey-Campanato Spaces and classical operators

Prof. Alexey Karapetyants¹

¹Institute for Mathematics, Mechanic and Computer Sciences, Southern Federal University, Russia
Email: karapetyants@gmail.com


Course requirements: Basic knowledge of Functional analysis, Real analysis and Linear operator theory

Examination and grading:

SSD: MAT/05

Aim: Morrey spaces are widely used in applications to regularity properties of solutions to PDE’s. We overview known and recently obtained results on Morrey-Campanato spaces with respect to the properties of the spaces themselves, and also, we overview the study of classical operators of harmonic analysis in these spaces. We also proceed with some generalizations and modifications.

Course contents:


Bibliography: (the full list of sources will be given to the students in a separate file before the beginning of the course with recommendations).


Introduction of Shimura Varieties

Prof. Elena Mantovan

1Department of Mathematics, California Institute of Technology
Email: mantovan@caltech.edu

Timetable: 16 hrs, First lecture on October 22, 2019, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: basic topology and basic geometry.

Examination and grading:

SSD: MAT/03

Aim:

Course contents:
We will give an introduction to the theory of Shimura varieties, and to the arithmetic theory of automorphic forms, focusing on examples. Among the aims of this course, it is to explain the role of the arithmetic theory of Shimura varieties in the classical Langlands program, that is in the study of the connections between automorphic forms and Galois representations.

Topics will include:

2. Complex theory: Double coset spaces, Hermitian symmetric domains, bounded realizations, holomorphic automorphic forms.
5. Good reduction: Abelian varieties over finite fields, Barsotti–Tate groups, Serre–Tate theory. Geometry of the special fibers of Shimura varieties: Newton polygons strata, Ekedahl–Oort strata, Oort leaves.

References:

Introduction to Floer Homology

Marco Mazzucchelli

CNRS, École Normale Supérieure de Lyon, UMPA, France
Email: marco.mazzucchelli@ens-lyon.fr

Timetable: 12 hrs. First lecture on February 28, 2020, 14:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: The students attending this course are required to know the basics of functional analysis (Banach and Hilbert spaces), differential geometry and topology (manifolds, vector fields, differential forms, vector bundles, Riemannian metrics, critical points of a smooth map), and some symplectic geometry (symplectic forms, Hamiltonian vector fields).

Examination and grading:

SSD: MAT/

Aim:

Course contents (tentative):

- **Lecture 1:** Crash course in algebraic topology: singular homology and co-homology, De Rham cohomology.
- **Lecture 2:** The Morse homology theorem.
- **Lecture 3:** Variational principle for Hamiltonian periodic orbits, action spectrum, the Conley-Zehnder index.
- **Lecture 4:** Construction of the Floer homology groups for aspherical manifolds I.
- **Lecture 5:** Construction of the Floer homology groups for aspherical manifolds II, proof of the Arnold conjecture on the fixed points of generic Hamiltonian diffeomorphisms.
- **Lecture 6:** Bott’s iteration formula for the Conley-Zehnder index, proof of the Conley conjecture on the periodic points of generic Hamiltonian diffeomorphisms of aspherical manifolds. Bonus arguments: Floer homology for monotone manifolds, products in Floer homology, spectral invariants, symplectic homology, etc.

References:


Positivity Principles and decay of Solutions in Semilinear Elliptic Problems

Vitaly Moroz

Swansea University, UK
Email: v.moroz@swansea.ac.uk


Tentative timetable (subject to minor variations):
Tuesday June 23, 2020, 11:20, Room P4
Wednesday June 24, 2020, 11:20, Room P4
Thursday June 25, 2020, 11:20, Room P4
Friday June 26, 2020, 11:20, Room P4
Friday June 26, 2020, 14:30, Room 2BC/30 (Torre Archimede)
Monday June 29, 2020, 14:30, Room 2BC/30 (Torre Archimede)
Monday June 29, 2020, 15:30, Room 2BC/30 (Torre Archimede)

Course requirements: Students should have a basic knowledge in partial differential equations.

Examination and grading:

SSD: MAT/05

Aim: The purpose of the course is to introduce two core but not widely known ideas of the linear elliptic theory, namely Allegretto–Piepenbrink positivity principle and Phragmén-Lindelöf comparison principle, and to show how these two fundamental principles provide a powerful tool in the analysis of the structure of positive solutions for large classes of semilinear elliptic equations. The course will consist of the core part, delivered in 5 lectures during the Mini-courses in Mathematical Analysis 2020, and additional 3 lectures containing advanced material.

Course contents:

Mini-course material – 5 lectures

Lecture 1: Allegretto–Piepenbrink positivity principle for linear Schrödinger operators and some corollaries: optimal and improved Hardy inequalities, Barta type inequality, torsion function estimate.

Lecture 2: Phragmén-Lindelöf comparison principles for linear Schrödinger operators, large and small positive solutions, admissible decay for sub- and super-solutions; concept of a weak and strong perturbation potentials.

Lecture 3: Nonlinear Liouville theorems for semilinear elliptic equations in unbounded domains, Serrin’s critical exponent(s), fast and slow decay solutions.

*Please, note that the scheduled time is not adjustable as the lectures are part of the Workshop “Minicorsi di Analisi Matematica”


Advanced material – 3 lectures


Lecture 2: Riesz potentials and their basic properties. Decay estimates and localization principle for the Riesz potentials.


References:


Topics in the representation theory of infinite-dimensional Lie algebras

Prof. Ivan Penkov

1 Jacobs University Bremen, Germania
Email: i.penkov@jacobs-university.de

INdAM Visiting Professor

Timetable: 16 hrs. First lecture October 24, 2019, 14:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: solid knowledge of linear algebra and knowledge of Lie theory at beginner’s level.

Examination and grading: oral examination on the topics covered during the course

SSD: MAT/02

Aim: The course aims at providing an introduction to the theory of locally finite Lie algebras and their representations. After a brief introduction into Lie algebra theory, the course will concentrate on the structure and representation theories of the three simple finitary infinite-dimensional Lie algebras $\mathfrak{sl}_\infty$, $\mathfrak{o}_\infty$, $\mathfrak{sp}_\infty$. If time permits, an application to the boson-fermion correspondence will be presented.

Course contents:
Synopsis: Finite-dimensional simple Lie algebras and their representations (3 hours), the Lie algebras $\mathfrak{sl}_\infty$, $\mathfrak{o}_\infty$, $\mathfrak{sp}_\infty$ - introduction (1 hour), Cartan, Borel and parabolic subalgebras (3 hours), weight representations (2 hours), simple modules with a highest weight and without highest weight (2 hours), bounded weight modules (1 hour), tensor modules and boson-fermion correspondence (4 hours).

References:

3. I. Dimitrov, I. Penkov, Locally semisimple and maximal subalgebras of the finitary Lie algebras $\mathfrak{gl}_\infty$, $\mathfrak{sl}_\infty$, $\mathfrak{o}_\infty$ and $\mathfrak{sp}_\infty$, Journal of Algebra 322 (2009), 2069-2081.

Contramodules and their applications to tilting theory and Enochs’ conjecture

Prof. Leonid Positselki

1 Russian Academy of Sciences
Email: posic@mccme.ru

Timetable: 16 hrs. First lecture on November 21, 2019, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: knowledge of basic concepts of category theory and homological algebra, such as abelian categories and their derived categories, will be largely presumed.

Examination and grading:

SSD:

Aim:

Course contents:
Contramodules are module-like algebraic structures with infinite summation operations subject to natural axioms. For any infinitely generated n-tilting (or infinity-tilting) module, the heart of the related tilting t-structure is the category of contramodules over the topological ring of endomorphisms of the tilting module. The course will start with a discussion of comodules and contramodules over coalgebras and proceed to the tilting-cotilting correspondence, contramodules over topological rings, topologically semisimple and topologically perfect topological rings, and a discussion of the contramodule-based approach to the Enochs conjecture about covers and direct limits in module categories.
Introduction to Optimal Control Theory

Franco Rampazzo

1 Dipartimento di Matematica “Tullio Levi-Civita”, Università di Padova
Email: rampazzo@math.unipd.it

Timetable: 16 hrs. First lecture on January 27, 2020, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: Calculus for many variables and basic tools of Lebesgue measure theory. The needed functional analysis will be recalled during classes, so it not a prerequisite.*

Examination and grading: The final exams will consists either of a standard oral questioning on the main parts of the program or of a shortened recognition of the program togetrher with the dissertation on a research pa- per previously studied by the student.

SSD: MAT/05

Aim:
This course aims to provide the student with some basic tools of Optimal Control Theory. The latter generalizes Calculus of Variations to the case when the state trajectories are subject to differential equations with control parameters. Besides being a crucial for many mathematical subjects (e.g. Differential Geometry, Hamilton-Jacobi Pde’s, Mean Field Games, Differential games) Optimal Control Theory is quite motivated by applications like Aerospace Engeneering, Medicine, Economics, Ecology.

Course contents:

*Students who aim to a more detailed preparation to the course may follow the lectures by Prof. Rampazzo within the math undergraduate course “Analisi Superiore”. Contact him if interested.
An introduction to Supergravity in 11-dimensions

Dr. Andrea Santi¹

¹Department of Mathematics; University of Padova
Email: andrea.santi@math.unipd.it

Timetable: 16hrs. May 2019, Torre Archimede, Room 2BC/30.

Course requirements: linear algebra, group theory and very basic knowledge in differential geometry. I do not assume any knowledge of physics: all relevant definitions will be given along.

Examination and grading: written homework.

SSD: MAT/02-03, MAT/07

Aim: The purpose of the course is to give a gentle introduction to the geometric and Lie algebraic aspects of 11-dimensional supergravity and to motivate the study of its supersymmetric backgrounds.

Course contents: We will start with the fundamental objects of Riemannian and Lorentzian geometry, leading to the classical Einstein’s and Maxwell’s equations. We will discuss the basics of G-structures and Riemannian geometry, with a particular attention on the connection defining the so-called Killing transport and then consider special PDEs of geometric origin on spinor fields on Riemannian manifolds. In the second part of the course, we introduce 11-dimensional supergravity, the most important theory where Einstein’s General Relativity is combined with supersymmetry. We will discuss the construction of a Lie superalgebra generated from spinor fields satisfying a certain PDE and the homogeneity theorem, which states that supergravity backgrounds preserving more than half of the supersymmetry are homogeneous. Considerable emphasis will be placed on clarifying the relevant definitions, along with examples.

Course outline:

1. Differential calculus on manifolds
   (Tensor fields, Lie derivative, linear connections, curvature, Riemannian and Lorentzian metrics, Levi-Civita connection, Ricci and scalar curvatures, Einstein’s equations, Maxwell’s equations)

2. G-structures
   (Basic definitions and examples, symmetries of the integrable model, the automorphism group of a G-structure and Sternberg’s Theorem, Poincaré algebra)

3. Riemannian geometry
   (Killing vector fields and Killing transport, space forms, Killing algebras as filtered deformations)

4. Spin geometry
   (Clifford algebras, basics of spin geometry, spinor fields satisfying special PDEs, a toy model: extended Killing algebras)
5. Supersymmetry and Detour on Lie superalgebra theory
   (physical motivation of supersymmetry, Kac’s classification of simple Lie superalgebras, supertranslation algebra and Poincaré superalgebra)

6. Supergravity in 11-dimensions
   (Basic definitions, supergravity Killing spinors, examples: maximally supersymmetric backgrounds and brane solutions, homogeneity theorem, Killing superalgebras, Killing supertransport, highly supersymmetric backgrounds).
The Master Degree (Laurea Magistrale) in Mathematics of this Department offers many courses on a wide range of topics, in Italian or in English. The PhD students are encouraged to follow the parts of such courses they think are useful to complete their basic knowledge in Mathematics. In some cases this activity can receive credits from the Doctoral school, upon recommendation of the supervisor of the student. Since the courses at the Master level are usually less intense than those devoted to graduate students, the number of hours given as credits by our Doctorate will be less than the total duration of the course. Some examples of courses that receive such credits, unless the student already has the material in his background, are the following.
Soft Skills

1. Maths information: retrieving, managing, evaluating, publishing
Maths information: retrieving, managing, evaluating, publishing

Abstract: This course deals with the bibliographic databases and the resources provided by the University of Padova; citation databases and metrics for research evaluation; open access publishing and the submission of PhD theses and research data in UniPd institutional repositories.

Language: The Course will be held in Italian or in English according to the participants

Timetable: 4 hrs – March 2, 2020, 09:00 (2 hrs), March 9, 2020, 09:00 (2 hrs), Room 2BC/30
Other courses suggested to the students

The students are encouraged to follow also courses outside Padova if they are useful for their training to research, in accordance with their supervisor. Parts of such course can be counted in fulfilment of their duties, provided the student passes an exam. The number of hours recognised as credits will be decided by the Coordinator after hearing the supervisor. Some examples of courses that receive such credits are the following.
Courses in collaboration with the Doctoral School on “Information Engineering”

https://www.dei.unipd.it/node/2399

for the Class Schedule see on pag. 6 of the DEI’s Catalogue at the address above

The courses with * contain material that should be already known to a graduate student in Mathematics. Therefore the number of hours given as credits by our Doctorate will be less than the total duration of the course. The exact number of credits will be determined by the Coordinator after hearing the student and, if necessary, his/her supervisor.