Introduction to Quantum Groups

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Timetable: 24 hrs., Apr/May 2020, Torre Archimede, Room 2BC/30.

Course requirements:

Examination and grading:

SSD: MAT/02-03

Aim:

Quantum groups arose at first in connection with problems in statistical mechanics and are closely related to conformal field theory. Moreover, they have applications to many different areas of mathematics, e.g. knot theory and topology, the study of the absolute Galois group of \( \mathbb{Q} \), representation theory of algebraic groups in characteristic \( p \), Poisson-Lie groups, the theory of \((q)\)- special functions. They have also served as a rich source of examples in non-commutative geometry. For these reasons, the theory of quantum groups may be of interest to mathematicians with expertise in any of the following: algebra, number theory, geometry, topology, mathematical physics, analysis. The intent of this course is to offer an introduction to quantum groups and survey its diverse applications. Being aimed at a general audience, its prerequisites are limited to the basics of linear algebra and elementary notions of topology. Any other needed notion will be introduced through examples or formal definition.

Course contents: (Tentative)

1. History and Motivations, applications. Basic notions of representation theory.
2. The Lie algebra \( \mathfrak{sl}_2 \) and its representations (characteristic zero and modular). The universal enveloping algebra \( U(\mathfrak{sl}_2) \) of \( \mathfrak{sl}_2 \).
3. The quantized enveloping algebra \( U_q(\mathfrak{sl}_2) \) for \( q \) generic and \( q \) a root of unity.
5. Quantized universal enveloping algebras. PBW theorem. Specializations and its center
8. Quasi-Hopf algebras and applications to the absolute Galois group of \( \mathbb{Q} \).
10. Representations of Quantized universal enveloping algebras for generic \( q \).

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