Introduction to Floer Homology

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Timetable: 12 hrs. THE COURSE IS TEMPORARILY SUSPENDED. THE TIMETABLE OF THE LECTURES WILL BE COMMUNICATED AS SOON AS POSSIBLE

Course requirements: The students attending this course are required to know the basics of functional analysis (Banach and Hilbert spaces), differential geometry and topology (manifolds, vector fields, differential forms, vector bundles, Riemannian metrics, critical points of a smooth map), and some symplectic geometry (symplectic forms, Hamiltonian vector fields).

Examination and grading:

SSD: MAT/

Aim:

Course contents (tentative):

- **Lecture 1:** Crash course in algebraic topology: singular homology and co-homology, De Rham cohomology.
- **Lecture 2:** The Morse homology theorem.
- **Lecture 3:** Variational principle for Hamiltonian periodic orbits, action spectrum, the Conley-Zehnder index.
- **Lecture 4:** Construction of the Floer homology groups for aspherical manifolds I.
- **Lecture 5:** Construction of the Floer homology groups for aspherical manifolds II, proof of the Arnold conjecture on the fixed points of generic Hamiltonian diffeomorphisms.
- **Lecture 6:** Bott's iteration formula for the Conley-Zehnder index, proof of the Conley conjecture on the periodic points of generic Hamiltonian diffeomorphisms of aspherical manifolds. Bonus arguments: Floer homology for monotone manifolds, products in Floer homology, spectral invariants, symplectic homology, etc.

Refereces:

- M. Audin, M. Damian, Morse theory and Floer homology, Universitext. Springer, London; EDP Sciences, Les Ulis, 2014. xiv+596 pp. ISBN: 978-1-4471-5495-2; 978-1-4471-5496-9; 978-2-7598-0704-8.
- A. Banyaga, D. Hurtubise, Lectures on Morse Homology, Lectures on Morse homology. Kluwer Texts in the Mathematical Sciences, 29. Kluwer Academic Publishers Group, Dordrecht, 2004. x+324 pp. ISBN: 1-4020-2695-1.

- D. Salamon, Lectures on Floer homology, Symplectic geometry and topology (Park City, UT, 1997), 143-229, IAS/Park City Math. Ser., 7, Amer. Math. Soc., Providence, RI, 1999.
- D. Salamon, E. Zehnder, Morse theory for periodic solutions of Hamiltonian systems and the Maslov index, Comm. Pure Appl. Math., 45 (1992), 1303-1360
- M. Schwarz, Morse homology, Progress in Mathematics, 111. Birkhauser Verlag, Basel, 1993. x+235 pp. ISBN: 3-7643-2904-1