Introduction to Floer Homology

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Timetable: 12 hrs. First lecture on February 28, 2020, 14:00 (dates already fixed, see calendar), Torre Archimede, Room 2BC/30.

Course requirements: The students attending this course are required to know the basics of functional analysis (Banach and Hilbert spaces), differential geometry and topology (manifolds, vector fields, differential forms, vector bundles, Riemannian metrics, critical points of a smooth map), and some symplectic geometry (symplectic forms, Hamiltonian vector fields).

Examination and grading:

SSD: MAT/

Aim:

Course contents (tentative):

• Lecture 1: Crash course in algebraic topology: singular homology and co-homology, De Rham cohomology.
• Lecture 2: The Morse homology theorem.
• Lecture 3: Variational principle for Hamiltonian periodic orbits, action spectrum, the Conley-Zehnder index.
• Lecture 4: Construction of the Floer homology groups for aspherical manifolds I.
• Lecture 5: Construction of the Floer homology groups for aspherical manifolds II, proof of the Arnold conjecture on the fixed points of generic Hamiltonian diffeomorphisms.
• Lecture 6: Bott’s iteration formula for the Conley-Zehnder index, proof of the Conley conjecture on the periodic points of generic Hamiltonian diffeomorphisms of aspherical manifolds. Bonus arguments: Floer homology for monotone manifolds, products in Floer homology, spectral invariants, symplectic homology, etc.

References:

