Positivity Principles and decay of Solutions in Semilinear Elliptic Problems

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Tentative timetable (subject to minor variations):
Tuesday June 23, 2020, 11:20, Room P4
Wednesday June 24, 2020, 11:20, Room P4
Thursday June 25, 2020, 11:20, Room P4
Friday June 26, 2020, 11:20, Room P4
Friday June 26, 2020, 14:30, Room 2BC/30 (Torre Archimede)
Monday June 29, 2020, 14:30, Room 2BC/30 (Torre Archimede)
Monday June 29, 2020, 15:30, Room 2BC/30 (Torre Archimede)

Course requirements: Students should have a basic knowledge in partial differential equations.

Examination and grading:

SSD: MAT/05

Aim: The purpose of the course is to introduce two core but not widely known ideas of the linear elliptic theory, namely Allegretto–Piepenbrink positivity principle and Phragmén-Lindelöf comparison principle, and to show how these two fundamental principles provide a powerful tool in the analysis of the structure of positive solutions for large classes of semilinear elliptic equations. The course will consist of the core part, delivered in 5 lectures during the Mini-courses in Mathematical Analysis 2020, and additional 3 lectures containing advanced material.

Course contents:

Mini-course material – 5 lectures

Lecture 1: Allegretto–Piepenbrink positivity principle for linear Schrödinger operators and some corollaries: optimal and improved Hardy inequalities, Barta type inequality, torsion function estimate.

Lecture 2: Phragmén-Lindelöf comparison principles for linear Schrödinger operators, large and small positive solutions, admissible decay for sub–and super–solutions; concept of a weak and strong perturbation potentials.

Lecture 3: Nonlinear Liouville theorems for semilinear elliptic equations in unbounded domains, Serrin’s critical exponent(s), fast and slow decay solutions.

*Please, note that the scheduled time is not adjustable as the lectures are part of the Workshop “Minicorsi di Analisi Matematica”
**Lecture 4:** classification of singularities of semilinear elliptic equations, Keller–Osserman bound, removable singularities.

**Lecture 5:** boundary blow-up solutions of semilinear elliptic equations, global Keller–Osserman bound, classification of boundary blow-up solutions.

**Advanced material – 3 lectures**

**Lecture 1:** Nonlinear elliptic equations with nonlocal interactions: physical background, attractive and repulsive interactions. Choquard type equations.

**Lecture 2:** Riesz potentials and their basic properties. Decay estimates and localization principle for the Riesz potentials.

**Lecture 3:** Nonlocal Allegretto–Piepenbrink positivity principle. Liouville theorems and admissible asymptotic decay of positive solutions to Choquard equations.

**References:**


