An Introduction to moduli spaces

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Timetable: hrs. 24. First lecture on December 2, 2020, 11:00 (dates already fixed, see calendar), Torre Archimede, Room 1BC/50.

Course requirements: Basic notions of geometry, as provided by a Mathematics degree. This course is open to any PhD student. Depending on the students' background, I will include a review of useful notions from geometry and topology.

Examination and grading:

SSD: MAT/02

Aim:

Course contents:
A moduli space is a space parametrizing all possible objects of a certain fixed type. A classical example is the following: let us fix a compact oriented surface $\sum$. Then the space of all possible complex structures on $\sum$ is the moduli space $\mathcal{M}$ of genus g Riemann surfaces, where g is the topological genus of $\sum$. By construction, the points of $\mathcal{M}_g$ correspond to the isomorphism classes of Riemann surfaces of genus g. This kind of construction generalizes to many other classification problems. In good cases, moduli spaces will turn out to be complex manifolds or varieties. However, in most cases the moduli space will not be truly a manifold, but rather a mild generalization of it, called an *orbifold*. Although the construction of moduli spaces originates in algebraic geometry, moduli spaces themselves are of interest also in other areas of mathematics, such as other areas of geometry, topology, group theory, analysis and mathematical physics. In this course, I would like to present the basic ideas and formalism underlying the concept of moduli space, along with some main examples of interdisciplinary interest. Besides the moduli space of Riemann surfaces, interesting examples with applications in different fields include:

- mirror symmetry, a construction from theoretical physics that predicts that there exist pairs of topological spaces $(X; X^*)$ such that the moduli space of complex structures on X is isomorphic to the moduli space of symplectic structures on X^*, and vice versa;
- modular curves in number theory, which are moduli spaces parametrizing elliptic curves with additional structures.

References: (tentative list)

- Kock, Joachim; Vainsencher, Israel. An invitation to quantum cohomology. Kontsevich's formula for rational plane curves. Progress in Mathematics, 249. Birkhäuser Boston, Inc., Boston, MA, 2007. xiv+159 pp.

- Newstead, P. E. Introduction to moduli problems and orbit spaces. Tata Institute of Fundamental Research Lectures on Mathematics and Physics, 51. Tata Institute of Fundamental Research, Bombay; by the Narosa Publishing House, New Delhi, 1978. vi+183 pp.
- Cox, David A.; Katz, Sheldon. Mirror symmetry and algebraic geometry. Mathematical Surveys and Monographs, 68. American Mathematical Society, Providence, RI, 1999. xxii+469 pp.
- Harris, Joe; Morrison, Ian. Moduli of curves. Graduate Texts in Mathematics, 187. Springer-Verlag, New York, 1998. xiv+366 pp.