Campanato Spaces and their Applications to Regularity of Solutions to PDEs

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Timetable: 8 hrs., Spring 2021, Torre Archimede, Room 2BC/30.

Course requirements: standard undergraduate background in Mathematical Analysis

Examination and grading: oral exam

SSD: MAT/05

Course contents:
In his seminal papers published in 1963 Sergio Campanato introduced the spaces that nowadays are named after him. These spaces play an important role in regularity theory and harmonic analysis. The classical regularity approach was based on the singular integrals theory approach introduced by A.P. Calderón and A. Zygmund who proved in particular regularity in $L^p$ for solutions to elliptic or parabolic equations, see [?]. J. Nash [?] used this approach to solve the XIX Hilbert problem and proved Hölder regularity for solutions to elliptic and parabolic equations with $L^\infty$ coefficients. One year before, E. De Giorgi [?] proved the same result using a functional space approach. S. Campanato proved that De Giorgi’s approach works also in the case of equations with mildly regular coefficients (i.e., continuous or Hölder continuous coefficients) recovering the Calderon-Zygmund results, see [?, ?].

The focus of this PhD class is on Campanato’s approach. The lecture course will be organized as follows. Lectures I and II: definition of Campanato spaces, their embedding in the space of Hölder continuous functions, preliminary lemmata; Lectures III and IV: estimates in $L^p$ for solutions to elliptic operators; Lectures V and VI: estimates in Hölder classes for solutions to elliptic operators; Lectures VII and VIII: proof of De Giorgi regularity result.

Bibliography: