# Signatures in finance: life, death, and miracles

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Timetable: 12 hrs. Torre Archimede.

- 5.9: 10:00 11:30 Room 2BC30,
- 6.9: 10:00 11:30 Room 2BC30,
- 7.9: 10:00 11:30 Room 2BC30,
- 19.9: 10:00 11:30, 14:00-15:30 Meeting Room 702,
- 20.9: 10:00 11:30, 14:00-14:45 Meeting Room 702.
- 21.9: 10:00 11:30 Meeting Room 702

Course requirements: Probability and Stochastic Calculus (basic)

### Examination and grading: seminar

### **SSD:** MAT/06, SECS-S/06

**Aim:** This course aims at introducing the signature of a stochastic process, to then focus on recent development and application to Mathematical Finance, with a special emphasis on numerical aspects relative to its computation.

### **Course contents:**

Signature methods represent a non-parametric way for extracting characteristic features from time series data which is essential in machine learning tasks. This explains why these techniques become more and more popular in Econometrics and Mathematical Finance. Indeed, signature based approaches allow for data-driven and thus more robust model selection mechanisms, while first principles like no arbitrage can still be easily guaranteed.

In this course we shall therefore focus on the use of signature as universal linear regression basis of continuous functionals of paths for financial applications. We first give an introduction to continuous rough paths and show how to embed continuous semimartingales into the rough path setting. Indeed our main focus lies on signature of semimartingales, one of the main modeling tools in finance. By relying on the Stone-Weierstrass theorem we show how to prove the universal approximation property of linear functions of the signature in appropriate topologies on path space. In view of calibration of financial models, we shall also point out in which situations the signature approximation can be tricky. To cover models with jumps we shall additionally introduce the notion of cadlag rough paths, Marcus signature and its universal approximation properties in appropriate Skorokhod topologies. In the financial applications that we have in mind one key quantity that one needs to compute is the expected signature of some underlying process. Surprisingly this can be achieved for generic classes of jump diffusions (with possibly path dependent characteristics) via techniques from affine and polynomial processes. More precisely, we show how the signature process of these jump diffusions can be embedded in the framework of affine and polynomial processes. These classes of processes have been – due to their tractability – the dominating process class prior to the new era of highly over-parametrized dynamic models. Following this line we obtain that, in generic cases, the infinite dimensional Feynman Kac PIDE of the signature process can be reduced to an infinite dimensional ODE either of Riccati or linear type. This then allows to get power series expansions for the expected signature and its Fourier-Laplace transform.

In terms of financial applications, we shall treat two main topics: stochastic portfolio theory and signature based asset price models.

In the context of stochastic portfolio theory we introduce a novel class of portfolios which we call linear path-functional portfolios. These are portfolios which are determined by certain transformations of linear functions of a collections of feature maps that are non-anticipative path functionals of an underlying semimartingale. As main example for such feature maps we consider signature of the (ranked) market weights. Relying on the universal approximation theorem we show that every continuous (possibly path-dependent) portfolio function of the market weights can be uniformly approximated by signature portfolios. Besides these universality features, the main numerical advantage lies in the fact that several optimization tasks like maximizing expected logarithmic utility or mean-variance optimization within the class of linear path-functional portfolios reduces to a convex quadratic optimization problem, thus making it computationally highly tractable. We apply our method to real market data and show generic out-performance on out-of-sample data even under transaction costs.

In view of asset price models we consider a stochastic process whose dynamics are described by linear functions of the time extended signature of a primary underlying process, which can range from a (market-inferred) Brownian motion or a Lévy process to a general multidimensional semimartingale. The framework is universal in the sense that classical models can be approximated arbitrarily well and that the model's parameters can be learned from all sources of available data by simple methods. We provide conditions guaranteeing absence of arbitrage as well as tractable option pricing formulas for so-called sig-payoffs, exploiting the polynomial nature of generic primary processes. One of our main focus lies on calibration, where we consider both time-series and implied volatility surface data, generated from classical stochastic volatility models and also from S&P 500 index market data. For both tasks the linearity of the model turns out to be the crucial tractability feature which allows to get fast and accurate calibrations results. We also consider an adaptation of the model that allows to price and calibrate VIX options fast and accurately.