Conciseness of group words in residually finite groups

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Timetable: 16 hrs. First lecture on November 15, 2021, 11:00, (dates already fixed, look at Calendar of Activities on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC/30

Course requirements: Basic knowledge of Algebra

Examination and grading: Oral examination

SSD: MAT/02

Aim:

Course contents:

We will begin the course with a gentle introduction to the theory of infinite groups, with focus on residually finite groups and profinite groups. In the second part of the course we will discuss group-words in residually finite groups. A group-word $w = w(x_1, ..., x_r)$ is a non-trivial element of the free group on $x_1, ..., x_r$. We take an interest in the set of all $w$-values in a group $G$ and the verbal subgroup generated by it; they are $G_w = \{ w(g_1, ..., g_r) | g_1, ..., g_r \in G \}$ and $w(G) = \langle G_w \rangle$: The word $w$ is said to be concise in a class $C$ of groups if, for each $G$ in $C$ such that $G_w$ is finite, then also $w(G)$ is finite.

A conjecture proposed by Philip Hall in the 60s predicted that every word $w$ would be concise in the class of all groups, but almost three decades later the assertion was famously refuted by Ivanov. On the other hand, the problem for the class of residually finite groups remains open. A group is residually finite if and only if the intersection of all its normal sub-groups of finite index is trivial.

It is now well known that in many situations the results on residually finite groups are markedly different from the general case. In particular, this phenomenon is clearly illustrated by the solutions of Burnside problems. While the answer to the General Burnside Problem turned out to be negative, in the late 80s Zelmanov solved in the affirmative the Restricted Burnside Problem, which states that every residually finite group of finite exponent is locally finite. In recent years several new positive results about conciseness of words in residually finite groups were obtained.

We will present some of these results and a natural variation of the notion of conciseness in the class of profinite groups.