Doctoral Program in Mathematical Sciences
Department of Mathematics "Tullio Levi-Civita"
University of Padova

Doctoral Program in Mathematical Sciences

Catalogue of the courses 2021-2022

Updated June 27th, 2022

INTRODUCTION

This Catalogue contains the list of courses offered to the Graduate Students in Mathematical Sciences for the year 2021-2022.
The courses in this Catalogue are of three types.

1. Courses offered by the Graduate School (= Courses of the Doctoral Program)
2. Courses offered by one of its curricula.
3. Other courses of the following types: a) selected courses offered by the Master in Mathematics; b) selected courses offered by the PhD school in Information Engineering; c) selected courses offered by other PhD schools or other Institutions; d) reading courses.

(This offer includes courses taught by internationally recognized external researchers. Since these courses might not be offered again in the near future, we emphasize the importance for all graduate students to attend them.)
Taking a course from the Catalogue gives an automatic acquisition of credits, while crediting of courses not included in the Catalogue (such as courses offered by the Scuola Galileiana di Studi Superiori, Summer or Winter schools, Series of lectures devoted to young researchers, courses offered by other PhD Schools) is possible, but it is subject to the approval of the Executive Board. Moreover, at most one course of this type may be credited.
We underline the importance for all students to follow courses, with the goal of **broadening their culture in Mathematics**, as well as developing their knowledge in their own area of interest.

REQUIREMENTS FOR GRADUATE STUDENTS

Within the **first two years of enrollment (a half of these requirements must be fulfilled within the first year)** all students are required to

- pass the exam of at least four courses from the catalogue, among which at least two must be taken from the list of “Courses of the Doctoral Program”, while at most one can be taken among the list of “reading courses”
- participate in at least one activity among the “soft skills”
- attend at least two more courses

Students are warmly encouraged to take more courses than the minimum required by these rules, and to commit themselves to follow regularly these courses. At the end of each course the instructor will inform the Coordinator and the Secretary on the activities of the course and of the registered students.

Students **must register** to all courses of the Graduate School that they want to attend, independently of their intention to take the exam or not. We recommend to register as early as possible: the Graduate School may cancel a course if the number of registered students is too low. If necessary, the registration to a Course may be canceled.

<u>Courses for Master of Science in “Mathematics”</u>
Students have the possibility to attend some courses of the Master of Science in Mathematics and get credits. The recommendation that a student takes one of these courses must be made by the supervisor and the type of exam must be agreed between the instructor and the supervisor.

Courses attended in other Institutions and not included in the catalogue. Students activities within Summer or Winter schools, series of lectures devoted to young researchers, courses offered by the Scuola Galileiana di Studi Superiori, by other PhD Schools or by PhD Programs of other Universities may also be credited, according to whether an exam is passed or not; the student must apply to the Coordinator and crediting is subject to approval by the supervisor and the Executive board. We recall that at most one course not included in the Catalogue may be credited.

Seminars

a) All students, during the three years of the program, must attend the **Colloquia of the Department** and participate regularly in the Graduate Seminar ("**Seminario Dottorato**"), whithin which they are also required to deliver a talk and write an abstract.
b) Students are also strongly encouraged to attend the seminars of their research group.

HOW TO REGISTER AND UNREGISTER TO COURSES

The registration to a Course must be done online.
Students can access the **online registration form** on the website of the Doctoral Course http://dottorato.math.unipd.it/ (select the link Courses Registration), or directly at the address http://dottorato.math.unipd.it/registration/.
In order to register, fill the registration form with all required data, and validate with the command "Register". The system will send a confirmation email message to the address indicated in the registration form; please save this message, as it will be needed in case of cancellation.

Registration to a course implies the commitment to follow the course.

Requests of **cancellation** to a course must be submitted in a timely manner, and **at least one month before the course** (except for courses that begin in October and November) using the link indicated in the confirmation email message.

REQUIREMENTS FOR PARTICIPANTS NOT ENROLLED IN THE GRADUATE SCHOOL OF MATHEMATICS

The courses in this catalogue, although part of activities of the Graduate School in Mathematics, are open to all students, graduate students, researchers of this and other Universities.
For organization reasons, external participants are required to **communicate their intention** (loretta.dallacosta@unipd.it) to take a course at least two months before its starting date if the course is scheduled in January 2022 or later, and as soon as possible for courses that take place until December 2021.
In order to **register,** follow the procedure described in the preceding section.
Possible **cancellation** to courses must also be notified.

Courses of the Doctoral Program

Courses of the "Mathematics" area

11. Prof.Franco Rampazzo
    Controllability of families of smooth and non-smooth vector fields **M-11**

12. Prof. Richad Vinter
    Dynamic Optimization **M-12**

Courses of the “Computational Mathematics” area

1. Prof.ssa Beatrice Acciaio
   Causal optimal transport **POSTPONED TO THE NEXT YEAR**

2. Prof. Turgay Bayraktar
   Pluri-Potential Theory and Zeros of Random Polynomials **MC-2**

3. Prof. Oleg Davydov
   Mesh1ess Finite Difference Methods **MC-3**

4. Prof. Antoine Jacquier
   A smooth tour around rough models in finance (From data to stochastics to machine learning) **MC-4**

5. Prof.ssa Yuliya Mishura
   Stochastic differential equations involving fractional Brownian motion **MC-5**

6. Prof. Tiziano Vargiolu
   Topics in Stochastic Analysis **POSTPONED TO THE NEXT YEAR**

Courses offered within the Masters’s Degree in Mathematics

(The courses included in this section will count up to 16 hours in the total amount of students’exams)

1. Offered Courses **MD-1**

Soft Skills

1. Maths information: retrieving, managing, evaluating, publishing **SS-1**

2. Writing a CV for academic positions **on April 29th, 2022, 10:00, Room 2AB40**

3. Writing a post doc application **in preparation**

4. Active participation in events organized by the Department devoted to the popularization of mathematics, like Venetonight, Kidsuniversity and others.

Courses in collaboration with the Doctoral School on “Information Engineering”

(The courses included in this section will count up to 16 hours in the total amount of students’exams)

Calendar of activities on

`https://calendar.google.com/calendar/u/0/embed?src=fvsl9bgkbnhhkqp5mmqpiurn6c@group.calendar.google.com&ctz=Europe/Rome`

1. Prof. Giorgio Maria Di Nunzio
   Bayesian Machine Learning **DEI-1**
2. Prof. Lorenzo Finesso
   Statistical Methods **DEI-3**
3. Prof. Fabio Marcuzzi
   Computational Inverse Problems **DEI-4**
4. Prof. Gianluigi Pillonetto
   Applied Functional Analysis and Machine Learning **DEI-5**
5. Prof. Domenico Salvagnin
   Heuristics for Mathematical Optimization **DEI-6**
6. Prof. Luca Schenato
   Applied Linear Algebra **DEI-7**
7. Prof. Gian Antonio Susto
   Elements of Deep Learning **DEI-9**

Courses of the Doctoral Program

Random Graphs and Networks

Prof. Alessandra Bianchi[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universiṭi Padova*
Email bianchi@math.unipd.it

Timetable: 24 hrs. First lecture on January 25, 2022, 11:00 (date already fixed, see the Calendar of activities at https://dottorato.math.unipd.it/calendar).

Location: The course will be held in presence at the Math. Departments of Padova according with the following calendar:
Jan 25, 26 - Feb 1, 2, 3, 8, 17 in Room 2BC60,
Jan 27 - Feb 10 in Room 1BC50,
Feb 9, 15, 16 in Room 2AB45,
and online by zoom: Meeting ID: 812 1156 9941 / Passcode: 294788
The link to access the meeting is also available at the webpage
https://www.math.unipd.it/ bianchi/PhD21.html where all the material of the course will be collected.

Course requirements: Basic knowledge of probability theory: discrete random variables, finite and countable probability spaces, convergence theorems (law of large number, central limit theorem).

Examination and grading: Seminar

SSD: MAT/06 Probability and Mathematical Statistics

Course contents:
Complex networks are attracting in recent years an increasing attention of the scientific community, due to the wide range of real-world situations in which they arise. Examples of networks emerge from the internet, from social relations and collaborations, from electrical power grids, or from biological and ecological environments. Important common features of all these examples appear from their large-scale behavior, where their share similar properties such as the "small words" and the "scale free" phenomena. Random graphs are the mathematical models that allow to analyze these large scale features; roughly, random graphs can be described as random variables taking values on a set of graphs, hence well suited to capture both probabilistic and combinatorial aspects of the real-world networks listed above. The course will focus on different classes of random graphs. We will start from the definition of the Erdos- Rényi random graph, one of simplest model one could think of. Despite its simplicity, this model presents relevant and unforeseen large-scale features that will be discussed along the course, including an interesting phase transition related to presence of a giant connected component. Keeping in mind the properties of real networks, we will then introduce and discuss three different families of random graphs: The Inhomogeneous Random Graph, the Configuration Model and the Preferential Attachment Model. For these random graphs we will prove precise asymptotic results for the degree distributions, and discuss their scale free and small

world behavior.

Lectures

1. Basic setting: graphs, trees, random graph setting, and main properties of the real-world networks.
2. Erdos-Renyi (ER) random graphs: Uniform and Binomial model; monotonicity and thresholds.
3. ER random graphs structure: trees containment, Poisson paradigm, largest component, connectivity.
4. Branching Processes: survival probability; total progeny; random walk perspective, duality principle.
5. Exploration process; largest component in subcritical regime.
6. Emergence of a giant component in ER- random graphs: Phase transition and behavior at criticality.
7. Inhomogeneous random graphs (IRG): degree sequence and scale-free property.
8. Configuration Model (CM): construction and simplicity probability. Uniform random graphs.
9. Multi-type branching process; local convergence of random graphs.
10. Phase transition and small world phenomenon in the IRG and in the CM.
11. Preferential Attachment Model (PAM): construction, scale free and small world properties.
12. Perspectives: stochastic models on random graphs.

Bibliografy:

- R. van der Hofstad. Random graphs and complex networks. Vol. 1. Cambridge Series in Statistical and Probabilistic Mathematics, [43]. Cambridge University Press, Cambridge, 2017. Available on the author webpage: https://www.win.tue.nl/ rhofstad/NotesRGCN.pdf
- R. van der Hofstad. Random graphs and complex networks. Vol. 2. To appear in Cambridge Series in Statistical and Probabilistic Mathematics. A preliminary version in pdf is available on the author webpage https://www.win.tue.nl/ rhofstad/NotesRGCNII.pdf
- A. Frieze, M. Karoński. Introduction to random graphs. Cambridge University Press, Cambridge, 2016. Available on the author webpage: https://www.math.cmu.edu/ af1p/BOOK.pdf
- R. van der Hofstad. Stochastic processes on random graphs. Lecture notes for the 47th Summer School in Probability Saint-Flour 2017. Available on the author webpage: https://www.win.tue.nl/ rhofstad/SaintFlour SPoRG.pdf

Derived categories in Algebraic Geometry and Representation Theory

D.ssa Luisa Fiorot[1], Prof. Ernesto Carlo Mistretta[2]

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[2] *Dipartimento di Matematica "Tullio Levi-Civita", Padova*
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Timetable: 24 hrs. First lecture on November 2nd, 2021, 14:30 (dates already fixed, see Calendar of the Activities), Torre Archimede, Room 2BC/30

Course requirements: algebra 1, geometry 1 and 2 of the first level degree in mathematics.

Examination and grading: please contact the teachers of the course by e-mail

SSD: MAT/02 MAT/03

Course contents:

- Introduction to projective varieties with examples. Sheaves in abelian groups over a projective variety. Global sections of a sheaf.
- Coherent and quasi-coherent sheaves. Vector bundles. Serre FAC.
- Check cohomology of sheaves, the group of Picard divisors and the group of Cartier divisors.
- Additive categories, abelian categories, functors, natural transformation, equivalence of categories with examples from algebra and algebraic geometry.
- The functor of global sections. Functors left/right exact. Tensor product and Hom functors.
- Computation of cohomology groups using injectives. Triangulated and derived categories with examples coming from algebra and algebraic geometry. Derived functors.
- Time permitting: Fourier-Mukai transform. Beilinson theorem: derived equivalence between the derived category of coherent sheaves of the projective line versus the derived category of modules over the path algebra of the Kronecker quiver.

Modern Optimization Methods

Prof. Francesco Rinaldi[1]

[1] Dipartimento di Matematica "Tullio Levi-Civita", Padova
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Timetable: 24 hrs. First lecture on May 2, 2022, 10:30 (dates already fixed, see Calendar of Activities at https://dottorato.math.unipd.it/calendar) Torre Archimede, Room 1BC45.

Course requirements: A basic knowledge of linear algebra, calculus and probability theory.

Examination and grading:

SSD: MAT/09

Aim:

Course contents:
In this course, we focus on some simple iterative optimization approaches that, thanks to the advent of the "Big Data era", have re-gained popularity in the last few years. We first review a bunch of classic methods in the context of modern real-world applications. Then, we discuss both theoretical and computational aspects of some variants of those classic methods. Finally, we examine current challenges and future research perspectives. Our presentation, strongly influenced by Nesterov's seminal book, includes the analysis of zeroth and first-order methods, stochastic optimization methods, randomized and distributed methods, projection-free methods. The theoretical tools considered in the analysis, together with the broad applicability of those methods, makes the course quite interdisciplinary and might be useful for PhD students in different areas (like, e.g., Analysis, Numerical Analysis, Operations Reearch, Probability and Mathematical Statistics).

Optimal Transport and Wasserstein Gradient Flows

Prof. Filippo Santambrogio[1]

[1] *Institut Camille Jordan, Université Claude Bernard - Lyon 1, Villeurbanne, FRANCE*
Email: santambrogio@math.univ-lyon1.fr

Timetable: 24 hrs. First lecture on April 11th, 2022, 09:00.

Location: The course will be held in presence and online according with the following calendar:
Apr 11, 12, 09:00-11:00, in Room 1BC45 (Prof. Davide Vittone),
plus lectures that will take place in 10 out of the following 12 slots:
Apr 20, 21, 10:30-12:30 and 14:00-16:00, in Room 2BC30, Apr 22, 10:30-12:30, in Room 2AB40, and 14:00-16:00, in Room 1BC45 (Prof. Filippo Santambrogio)
Apr 26, May 3, 10, 17, 24, 31 14:00-16:00 online (Prof. Filippo Santambrogio)

Course requirements: some functional analysis (for instances chapters 1, 3, 4, 8 and 9 of Brezis' book on functional analysis) and some notions of basic PDEs. However, the main required notions will be recalled during the course.

Examination and grading: Oral examination (on the content of the course, or presentation of a related research paper/subject, according to the preferences of each student)

SSD: MAT-05

Aim: With the first part of the course students will learn the main features of the theory of optimal transport; the second part will allow them to master more specialized tools from this theory in their applications to some evolution PDEs with a gradient flow structure

Course contents:
Monge and Kantorovich problems, existence of optimal plans, duality.
Existence of optimal maps, Brenier's theorem (optimal maps in the quadratic case are gradient of convex functions), connection with the Monge-Ampère equation.
Optimal transport for the distance cost. Wasserstein distances and their properties.
Curves in the Wasserstein spaces and relation with the continuity equation. Characterization of AC curves in the Wasserstein spaces
Geodesics in the Wasserstein spaces. Dynamic Benamou-Brenier formulation.
Introduction to gradient flows in metric spaces. The JKO minimization scheme for some parabolic equation.
Convergence of the JKO scheme for the Heat and Fokker-Planck equations.
Regularity estimates from the JKO scheme (Lipschitz, BV, Sobolev…).

Bibliography:
F. Santambrogio: *Optimal Transport for Applied Mathematicians*, Birkhauser (2015)
C. Villani: *Topics in Optimal Transportation,* American Mathematical Society (2003)
L. Ambrosio, N. Gigli, G. Savaré: *Gradient Flows in Metric Spaces and in the Space of Probability Measures*, Birkhauser (2005)
F. Santambrogio: {Euclidean, Metric, and Wasserstein} Gradient Flows: an overview, *Bulletin of Mathematical Sciences*, available online (2017).

An Introduction to Kinetic Theory and Boltzmann equation (Analysis, Mathematical Physics and applications)

Prof. Daniela Tonon[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Padova*
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Timetable: 24 hrs. First lecture on March 15, 2022, 09:00 (dates already fixed, see Calendar of Activities at https://dottorato.math.unipd.it/calendar) Torre Archimede, Room 2BC30.

Course requirements: Basic mathematical analysis.

Examination and grading:

SSD: MAT/05

Course contents:

We introduce kinetic theory, i.e. the theory that describes the behavior of rarefied gases at a mesoscopic scale, a scale that can be considered as in between the microscopic scale (where the gas is described as a set of a large number of particles) and the macroscopic one (where the gas is described as a continuum fluid). This mesoscopic description models the behavior of an average gas particle through statistical quantities, simplifying the detailed study of trajectories and preserving information on the physic quantities of the system. Starting form the classical free transport equation, we will describe the crucial role of the collisional operator in several different models that can be derivated from physical assumptions (Landau, Fokker-Planck, Boltzmann). In particular we will focus on the formal derivation of Boltzmann equation (that models hard spheres gases) and on the techniques used to cope with its particular, highly singular, collisional operator in the study of the Cauchy problem (cut-off theory, renormalized DiPerna and Lions solutions, velocity-averaging Lemmas, perturbative solutions and linearization). We will conclude the overview on kinetic theory dealing with the study of the Boltzmann equation in the more physically relevant case of bounded domains, considering several different boundary conditions such as in flow, specular reflection, bounce-back reflection and diffuse boundary conditions.

A particular emphasis will be given to the mathematical aspects as well as to the physical modeling and some numerical simulations.

Bibliografy:

- Cercignani, C., Mathematical Methods in Kinetic Theory, Springer-Verlag US 1969
- Cercignani, C., The Boltzmann Equation and Its Applications, Applied Mathematical Sciences book series, volume 67, Springer-Verlag New York Inc. 1988
- Cercignani, C., Illner, R. , Pulvirenti, M. The Mathematical Theory of Dilute Gases, Applied Mathematical Sciences book series, volume 106, Springer-Verlag New York Inc. 1994
- Villani, C., A review of mathematical topics in collisional kinetic theory, Notes available on www.cedricvillani.org

Courses of the "Mathematics" area

Basics on Hida Theory

Luca Dall'Ava[1], Maria Rosaria Pati[2]

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[2] *Dipartimento di Matematica "Tullio Levi-Civita", Universiṭi Padova*
Email: pati@math.unipd.it

Timetable: 8 hrs. in total. First lecture on March 18th, 2022, h14:15, Torre Archimede, SR701 (18/3, 22/4) and 2AB40.

Course requirements: Basic knowledge of algebraic number theory, commutative algebra, and complex analysis. Elementary knowledge of modular forms is required.

Examination and grading: Seminar talk about an advanced topic related to the course.

SSD: MAT/02, MAT/03.

Aim: Present the basics on the theory of families of modular forms, as developed by Hida.

Course contents: We present the notion of p-adic modular forms and provide the naive definition of the weight-space as given by Serre. Starting with the definition of ordinary modular forms, we then define families of ordinary modular forms and briefly state their relation with Hecke algebras. Time permitting, we examine the approach via Galois representations.

Bibliography:

1. Greenberg, R. and Stevens, G., *p-adic L-functions and p-adic periods of modular forms*, Invent. Math. 111 (1993), no. 2, 407–447.
2. Hida, H., *Elementary theory of L-functions and Eisenstein series*, London Mathematical Society Student Texts, 26. Cambridge University Press, Cambridge, 1993. xii+386.
3. Serre, J.-P., *Formes modulaires et fonctions zêta p-adiques*, Modular functions of one variable, III (Proc. Internat. Summer School, Univ. Antwerp, 1972), pp. 191–268, Lecture Notes in Math., Vol. 350, Springer, Berlin, 1973.
4. Wiles, A., *On ordinary λ-adic representations associated to modular forms*, Invent. Math. 94 (1988), no. 3, 529–573.

Convex integration: from isometric embeddings to Euler and Navier Stokes equations

Prof.ssa Sara Daneri[1]

[1] *Gran Sasso Science institute*
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Timetable: 16 hours, first lecture on May 3, 2022, 09:00 (dates already fixed, see Calendar of Activities at https://dottorato.math.unipd.it/calendar) Torre Archimede, Room 2BC30.

Course requirements: Very basic notions of ODE and PDE theory and of differential geometry (Riemannian manifold, length of curves).

Examination and grading: Seminar.

SSD: MAT/05 - Mathematical Analysis

Aim and Course contents: Convex integration, first introduced by Nash to prove nonuniqueness of C^1 isometric embeddings of Riemannian manifolds, turned out in the last ten years (starting from De Lellis and Székelyhidi) to be a very powerful tool to show nonuniqueness and flexibility of solutions (h-principle) in problems of fluid mechanics. Aim of the course is to explain different applications of the technique of convex integration to some of these problems. In particular, we will focus on the solution of major problems like the Onsager's conjecture on the existence of dissipative Hölder solutions to the Euler equations and the more recent proof of nonuniqueness for weak (non Leray) solutions to the 3D Navier Stokes equations. The first application of convex integration, namely that to the nonuniqueness of C^1 isometric embeddings of Riemannian manifolds, will also be covered.

The course should be particularly interesting for students in Mathematical Analysis, Differential Geometry and Mathematical Physics, in particular those interested in Fluid Mechanics.

Bibliografy:

- L. Székelyhidi "From isometric embeddings to turbulence" Lecture Notes avaliable online.
- S. Daneri "Convex integration: from isometric embeddings to Euler and Navier stokes equations" Lecture notes which will be given during the course.

Conciseness of group words in residually finite groups

Prof.ssa Eloisa Detomi[1], Prof. Pavel Shumyatsky[2]

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[2] *Department of Mathematics, University of Brasilia, Brasile*
Email: pavel@unb.br

Timetable: 16 hrs. First lecture on November 15, 2021, 11:00, (dates alreeady fixed, look at Calendar of Activities on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC/30

Course requirements: Basic knowledge of Algebra

Examination and grading: Oral examination

SSD: MAT/02

Aim:

Course contents:
We will begin the course with a gentle introduction to the theory of infinite groups, with focus on residually finite groups and profinite groups. In the second part of the course we will discuss group-words in residually finite groups. A group-word $w = w(x_1, ..., x_r)$ is a non-trivial element of the free group on $x_1, \vdots, x_r$. We take an interest in the set of all w-values in a group G and the verbal subgroup generated by it; they are $G_w = \{w(g_1, ..., g_r)|g_1...g_r \in G\}$ and $w(G) = \langle G_w \rangle$: The word w is said to be concise in a class $\mathcal{C}$ of groups if, for each G in $\mathcal{C}$ such that G_w is finite, then also $w(G)$ is finite.
A conjecture proposed by Philip Hall in the 60s predicted that every word w would be concise in the class of all groups, but almost three decades later the assertion was famously refuted by Ivanov. On the other hand, the problem for the class of residually finite groups remains open. A group is residually finite if and only if the intersection of all its normal sub-groups of finite index is trivial.
It is now well known that in many situations the results on residually finite groups are markedly different from the general case. In particular, this phenomenon is clearly illustrated by the solutions of Burnside problems. While the answer to the General Burnside Problem turned out to be negative, in the late 80s Zelmanov solved in the affirmative the Restricted Burnside Problem, which states that every residually finite group of finite exponent is locally finite. In recent years several new positive results about conciseness of words in residually finite groups were obtained.
We will present some of these results and a natural variation of the notion of conciseness in the class of profinite groups.

Geometry and Dynamics of Nonholonomic Mechanical Systems

Prof. Luis C. García-Naranjo[1]

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Timetable: 12 hrs. First lecture on February 8th, 2022, 14:00, (dates already fixed, see Calendar of Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2AB40.

Course requirements: basic knowledge in differential equations, differential geometry and classical mechanics.

Examination and grading: seminar.

SSD: MAT/07

Aim: In mechanics, a nonholonomic constraint is a restriction on the possible velocities of the system without restricting its possible configurations. An example of this is the possibility to manoeuvre a car into parallel parking despite the nonholonomic constraint that prohibits the lateral displacement of its wheels. The study of nonholonomic mechanics is challenging because the equations of motion do not come from a variational principle and, as a consequence, do not possess a Hamiltonian structure. Therefore, many of the properties of the solutions remain poorly understood and the subject remains an active field of research. The aim of the course is to give an introduction to the subject, touching on geometric and dynamical aspects, that will present the students to recent progress and open questions in the field.

Course contents: The course will begin by describing the underlying physical principles of the theory and the geometric structure of the equations of motion. We will then present numerous examples and classify them according to their symmetries and their dynamical behaviour. We will proceed to review some recent known results on the existence of invariants of the dynamics such as first integrals, invariant measures and Poisson/symplectic structures, and confront these results with the examples presented earlier and discuss open questions.

Bibliografy:

1. Lecture notes will be prepared and made accessible to the students before the course.
2. R. Cushman, J.J. Duistermaat and J. Snyaticki, Geometry of Nonholonomically Constrained Systems. (World Scientific, 2010).

Degenerations of abelian varieties

Lars Halle[1]

[1] *Università di Bologna, Dipartimento di Matematica, Piazza di Porta San Donato 5, Bologna.*
Email: larshalvard.halle@unibo.it

Timetable: 16 hrs. First lecture on March 22nd, 2022, 11:00 (dates already fixed, see on https://dottorato.math.unipd.it/calendar/), Torre Archimede, Room 2BC30.
Most of the lectures will be given in-class, but a couple of the lectures might be given via zoom.

Course requirements:

1. Algebraic geometry/scheme theory roughly at the level of Chapters 2 and 3 of Hartshorne's book.
2. Basic knowledge of abelian varieties.

Examination and grading: Seminar on a course related topic.

SSD: MAT/03

Course contents: The course will give an introduction to Néron models of abelian varieties. This theory forms an important tool in the study of abelian varieties, with numerous applications in arithmetic questions as well as in moduli theory.

Let R be a discrete valuation ring with field of fractions K and residue field k. If A is an abelian variety defined over K, there exists a canonical way to extend A to a group scheme $\mathcal{A}$ over R – the *Néron model* of A. The special fiber $\mathcal{A}_k = \mathcal{A} \times_R k$ is again a smooth commutative group scheme, but its geometric structure can be much more complicated than that of A. For instance, it might be non-proper and even disconnected. Informally, one can say that the shape of $\mathcal{A}_k$ reflects the arithmetic properties of A over the field K (or, in geometric terms, the way in which A degenerates at the closed point of $\mathrm{Spec} R$).

Central themes that will be discussed in the course are as follows.

- *Structure of the special fiber* $\mathcal{A}_k$. In particular, the Chevalley decomposition of the connected component of identity $\mathcal{A}_k^\circ$ of $\mathcal{A}_k$, and the group of connected components $\Phi(A) = \mathcal{A}_k/\mathcal{A}_k^\circ$.
- Grothendieck's *semi-abelian reduction theorem*, as well as applications/consequences of this result.
- Néron models of Jacobians of curves, and the link to degenerations/models of curves.
- Examples of applications to arithmetic questions for curves and abelian varieties.

Spaces and operators in complex analysis

Prof. Alexey Karapetyants[1]

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Email: karapetyants@gmail.com

Timetable: 10 hrs. First lecture on November 8th, 2021, 14:30 (dates already fixed, see Calendar of the Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC/30

Course requirements:

Examination and grading:

SSD: MAT/05

Aim: The function theory of holomorphic spaces has experienced several breakthroughs during the last five-ten years. Some new spaces of nonstandard growth were introduced and studied. This was possible during to the mixture of the methods of real and complex harmonic analysis involved in the study of such spaces and operators. However, the basics are still going back to the classical Bergman type and even Hardy spaces. Therefore, studying classical approach along with new trends will open a good perspective for the further development of the subject.

Course contents:

1. Classical spaces of holomorphic functions: Hardy, Bergman, Besov, Holder spaces.
2. Basic properties of the mentioned spaces: duality, interpolation, atomic decomposition.
3. Holomorphic spaces of nonstandard growth: variable exponent, Orlicz, Morrey, generalized Holder spaces.
4. Operators and transforms on these classical and new spaces: Bergman projection, more general operators, including potentials and operators of fractional integroodifferentiation, Berezin transform, composition operators, Toeplitz operators.
5. Recent advances: new classes of integral operator in complex analysis of Hausdorf, Hardy and Bergman type, some applications.

Abelian varieties

Remke Kloosterman[1]

[1] Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova
Email: klooster@math.unipd.it

Timetable: 16 hrs. First lecture on January 18, 2022, 09:00 (date already fixed, see the Calendar of activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC/30

Course requirements: Basic knowledge of algebraic varieties and holomorphic functions.

Examination and grading: Seminar on a course related topic.

SSD: MAT/03

Aim: Introduction to the theory of abelian varieties.

Course contents:
This course discusses abelian varieties. We will start with the theory of complex tori and discuss when tori are embeddable (Riemann conditions) and the theory of line bundles on complex tori. The second part discusses abelian varieties over arbitrary fields and discuss isogenies, dual abelian varieties, the Picard scheme of an abelian varieties and degenerations of abelian varieties.
The final lecture will discuss the theory of Jacobians of curves.

Bibliografy:

- O. Debarre, Complex Tori and Abelian Varieties.SMF/AMS Texts and Monographs , vol. 11. 2005
- S.J. Edixhoven, G. van der Geer, B. Moonen. Abelian varieties. Available at `https://www.math.ru.nl/~bmoonen/research.html`

Categorical aspects of realizability

Samuele Maschio[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita"- Università di Padova*
Email: maschio@math.unipd.it

Timetable: 12 hrs. First lecture on November 4th, 2021, 14:00 (dates already fixed, see Calendar of the Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC/30

Course requirements: Basic notions in category theory and logic.

Examination and grading: Examination will consist of a seminar based on research papers on the topic of the course.

SSD: MAT/01, MAT/02, MAT/04, INF/01

Aim: The aim of the course is to present the categorical account of realizability. Realizability was introduced by Kleene in 1945 in order to provide an interpretation of intuitionistic arithmetics. The notion of realizability is connected to that of computability and exploits crucially Universal Turing Machine Theorem. As shown starting from the 80s, realizability can be presented in a categorical setting as it was done by Hyland with his Effective Topos, which provides a categorical framework for constructive mathematics. Here we take into account this construction in the wider context of tripos theory and exact completions, and we will present some of the main properties of realizability toposes and their internal mathematics.

Course contents: Kleene realizability. Elementary toposes; first-order hyperdoctrines; triposes; tripos-to-topos construction; partial combinatory algebras; realizability toposes and the effective topos; arithmetics in the effective topos; set theory in the effective topos; regular and exact categories; ex/reg completion; the category of assemblies, ex/lex completion; the category of partitioned assemblies.

Bibliografy:

1. A.Carboni, A.; E.M.Vitale. Regular and exact completions. J. Pure Appl. Algebra 125 (1998), no. 1-3, 79–116
2. J.Frey. Characterizing partitioned assemblies and realizability toposes, Journal of Pure and Applied Algebra 223.5 (2019): 2000-2014
3. J. M. E. Hyland. The effective topos. The L.E.J. Brouwer Centenary Symposium (Noordwijkerhout, 1981), 165–216, Studies in Logic and the Foundations of Mathematics, 110, North-Holland, Amsterdam-New York, 1982.
4. J. M. E. Hyland; P.T.Johnstone; A.M.Pitts. Tripos theory. Math. Proc. Cambridge Philos. Soc. 88 (1980), no. 2, 205–231.
5. M.E.Maietti, G.Rosolini: Unifying Exact Completions. Appl. Categorical Struct. 23(1): 43-52 (2015)
6. J. van Oosten: "Realizability. An introduction to its categorical side". Studies in Logic and the Foundations of Mathematics, vol.152. Elsevier, 2008.
7. Streicher, T.: Realizability. Lecture notes.

Variational Methods in Elasticity

Prof. Alessandro Musesti[1]

[1] Dipartimento di Matematica e Fisica "N. Tartaglia", Università Cattolica del Sacro Cuore, Brescia
Email: alessandro.musesti@unicatt.it

Timetable: 18 hrs (6 lectures of 3 hrs each). First lecture on March 3rd, 2022, 14:00, (dates alreeady fixed, look at Calendar of Activities on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC/30.

Course requirements: Basics of real analysis and Lebesgue spaces. Basics of continuum mechanics.

Examination and grading: Oral examination with presentation of selected research topics

SSD: MAT/07 - MAT/05

Aim: The course aims at introducing the main techniques of the three dimensional Calculus of Variations and at studying some variational problems of Continuum Mechanics in the framework of nonlinear hyperelasticity.

Course contents:

1. Introduction to the Direct Method in the Calculus of Variations. Weak Semicontinuity and Coercitivity. Sobolev Spaces.
2. Convexity. Kinematics of continua. Hyperelasticity. Isotropy of the physical space.
3. Quasiconvexity, Polyconvexity, Rank-one Convexity.
4. An existence result by John Ball.
5. Isotropy, Transverse Isotropy, Orthotropy. Polyconvexity of Ogden Materials.
6. Application to Biological Materials.

References:

[1] P. Ciarlet, Mathematical Elasticity, v. I, North Holland, 1988.

[2] B. Dacorogna, Direct Methods in the Calculus of Variations, Springer, 2008.

[3] M. Gurtin, Topics in Finite Elasticity, SIAM, 1983.

Mathematical methods of Quantum Mechanics

Prof. Antonio Ponno[1], Prof. Lorenzo Zanelli[2]

[1] Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova
Email:ponno@math.unipd.it

[1] Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova
Email:lzanelli@math.unipd.it

Timetable: 16 hrs. First lecture on November 10, 2021, 11:00 (dates already fixed, see Calendar of the activities on https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC/30.

Course requirements: Basic notions about Hilbert spaces and related operators theory, spectral theory, Hamiltonian mechanics.

Examination and grading: Seminar

SSD: MAT/07 - Mathematical Physics

Aim: The target of this course is to introduce the study of Quantum Mechanics in a rigorous way, through its most recent mathematical tools. The final achievement is to outline some of the current and more important research lines from the viewpoint of Mathematical Physics.

Course contents:

1. Basic notions of Quantum Mechanics.
2. Pseudodifferential Operators (Weyl, Wick and anti-Wick).
3. Phase space Analysis, coherent states and semiclassical approximations.
4. Many body dynamics, mean field asymptotics towards Hartree equation.

References:

- F.A.Berezin, M. Shubin: The Schrödinger equation. Mathematics and its Applications, Springer (1991).
- M.Combescure, D.Robert: Coherent states and applications in Mathematical Physics, Springer (2012).
- G.Folland: Harmonic Analysis in Phase Space. Princeton University Press (1989).
- D.Griffiths, D.Schroeter: Introduction to Quantum Mechanics (3rd ed.) Cambridge University Press (2018).
- L.Erdös, B.Schlein, H-T.Yau: Derivation of the Gross-Pitaevskii equation for the dynamics of Bose-Einstein condensate. Annals of Mathematics, 172 (2010) pp 291-370.

Controllability of families of smooth and non-smooth vector fields

Franco Rampazzo

Dipartimento di Matematica "Tullio Levi-Civita", Universitá di Padova
rampazzo@math.unipd.it

Timetable: 16 hrs. First lecture on March 2nd, 2022, 11:00 (date already fixed, see the Calendar of activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC/30

Course requirements: Basic mathematical analysis.

Examination and grading: The exam will consist in the presentation of some previously assigned article or book chapter (of course the student must show a good knowledge of those issues taught during the course which are connected with the presentation.).

SSD: MAT/05 Mathematical Analysis

Aim: to make students aware of smooth and non-smooth controllability results and of some applications in various fields of Mathematics and of technology as well.

Course contemts:
Vector fields are basic ingredients in many classical issues of Mathematical Analysis and its applications, including Dynamical Systems, Control Theory, and PDE's. Loosely speaking, *controllability* is the study of the points that can be reached from a given initial point through concatenations of trajectories of vector fields belonging to a given family. Classical results will be stated and proved, using coordinates but also underlying possible chart-independent interpretation. We will also discuss the non smooth case, including some issues which involve Lie brackets of nonsmooth vector vector fields, a subject of relatively recent interest.

Bibliography: Lecture notes written by the teacher.

Dynamic Optimization

Richard Vinter[1]

[1] *Department of Electrical and Electronic Engineering, Faculty of Engineering, Imperial College, London*
Email: r.vinter@imperial.ac.uk

Timetable: 12 hrs. First lecture on September 13, 2021, 10:00 (dates already fixed see calendar),Torre Archimede, Room 1BC50 and online at
Zoom Meeting
https://unipd.zoom.us/j/86729567873?pwd=S3dqS2U1Y1hkSWFpTE9rbEpCWUFGZz09
Meeting ID: 867 2956 7873 - Passcode: Vinter21

Course requirements: There are no pre-requisites, but if the students have done an earlier course in control, they will find my lectures easier to understand.

Examination and grading: Oral examination

SSD: MAT/05

Aim: Dynamic optimization concerns optimization problems, in which we seek to minimize a functional over arcs that satisfy some kind of dynamic constraint. When this constraint takes the form of a controlled differential equation, such problems are known as optimal control problems. In Dynamic Optimization, the 'dynamic constraint' is allowed a broader interpretation, and is formulated as, say, a differential inclusion. Earlier applications of the theory were principally in aerospace (selection of flight trajectories in space missions) and chemical engineering. But now the field is recognised as having far wider application, in econometrics, resource economics, robotics and control of driverless vehicles, to name just a few areas. Dynamic optimization dates back, as a unified field of study, to the early 1950's, when two breakthroughs occured. One was the Maximum Principle, a set to necessary conditions for a control function to be optimal. The other was dynamic programming, which reduces the search for optimal controls to the solution of the Hamilton Jacobi equation (HJE). Early developments in the field relied on classical analysis and, even today, introductory courses on Optimal Control given based on traditional calculus. But, more recently, advances in the field have increasingly depended on new techniques of nonlinear analysis, which are referred to, collectively, as nonsmooth analysis. Nonsmooth analysis aims to give meaning to the 'derivative' of functions that are not differentiable in the classical sense and to tangent vectors to sets with nonsmooth boundaries. First order necessary conditions and Hamilton Jacobi theory continue to have a prominent role in the latest developments, but now interpreted in deeper and more insightful ways.

We begin the course by identifying deficiencies in the early theory and explaining why new analytical tools were needed to move the theory forward. We then introduce these tools (the main constructs of nonsmooth analysis and an accompanying calculus) and use them to derive optimality conditions (both first order necessary conditions and conditions related to the (HJE) equation). Our goal is to bring participants in the course 'up to speed', so that they can follow the latest literature, understand the underlying motivation and make future contributions.

Course contents:
Part I (Preliminaries) Dynamic Optimization (significance and illustrative examples)
Nonsmooth Analysis:
- Basic constructs (subdifferentials, normal cones, etc.)
- Subdifferential calculus
- The generalized mean value inequality
- Nonsmooth multiplier rules in Nonlinear Programming

Additional analytic techniques:
- Variational principles
- Compactness of trajectories
- Quadratic inf convolution
- Exact penalisation

Part II (First Order Necessary Conditions)
The maximum principle
The nonsmooth maximum principle
Necessary conditions for differential inclusion problems:
- The Generalized Euler Inclusion The Hamiltonian Inclusion
- Refinements to allow for pathwise state constraints.

Part III (Dynamic Programming)
Invariance:
- Weak invariance theorems
- Strong invariance theorems

Generalized solutions to the HJ equations
Links with viscosity solution concepts
Characterisation of the value function as generalized solution of (HJE)
Refinements to allow for pathwise state constraints

Part IV (Miscellaneous Topics)
Regularity of minimizers
Non-standard optimal control problems:
- discontinuous state trajectory problems
- problems involving time delay
- open problems and future directions.

Bibliografy:
I will provide a handout and will deliver slides for my lectures. I will aim to make my handout self-contained and there are no course texts. (The course will be based on a revision of by 2000 book 'Optimal Control', which I am currently working on.)

Courses of the “Computational Mathematics” area

Causal optimal transport

Prof. Beatrice Acciaio[1]

[1] *Department of Mathematics, ETH Zurich*
Email: beatrice.acciaio@math.ethz.ch

Timetable: POSTPONED TO THE NEXT YEAR

Course requirements: Probability and Stochastic Calculus (basic)

Examination and grading: TBD

SSD: MAT/06, SECS-S/06

Aim: This course aims at introducing the required basis on optimal transport, to then focus on recent development on causal optimal stopping theory, with applications to Mathematical Finance.

Course contents: TBD

Bibliography:

Pluri-Potential Theory and Zeros of Random Polynomials

Prof. Turgay Bayraktar[1]

[1] *Faculty of Engineering and Natural Sciences, Sabanci University ISTANBUL, TURKEY*
Email tbayraktar@sabanciuniv.edu

Timetable: 16 hrs. First lecture on .... , 2021, Torre Archimede, Room 2BC/30

Course requirements:

Examination and grading:

SSD:

Course contents:
The purpose of this mini-course is to introduce basic notions of complex potential theory in order to study statistics and asymptotic distribution of zeros of random polynomials. In the first part, we will cover central objects of interest of the modern weighted pluripotential theory in several complex variables, such as the pluricomplex Green functions, regular sets, equilibrium measures, Bernstein-Markov measures and Bergman functions. In the second part, we will focus on statistics of zeros of random polynomials and polynomial mappings.

Bibliografy:

[Bay16] T. Bayraktar. Equidistribution of zeros of random holomorphic sections. Indiana Univ. Math. J., 65(5):1759–1793, 2016.

[Bay17] T. Bayraktar. Asymptotic normality of linear statistics of zeros of random polynomials. Proc. Amer. Math. Soc., 145(7):2917–2929, 2017.

[Bay20] T. Bayraktar. Mass equidistribution for random polynomials. Potential Anal., 53 (2020), no. 4, 1403-1421.

[BCHM18] T. Bayraktar, D. Coman, H. Herrmann, and G. Marinescu. A survey on zeros of random holomorphic sections. Dolomites Res. Notes Approx., 11(4):1–19, 2018.

[BCM] T. Bayraktar, D. Coman, and G. Marinescu. Universality results for zeros of random holomorphic sections. Trans. Amer. Math. Soc., 373(6): 3765-3791, 2020.

[Bl05] T. Bloom. Random polynomials and Green functions. Int. Math. Res. Not., (28):1689–1708, 2005.

[Bl] T. Bloom and N. Levenberg. Random Polynomials and Pluripotential-Theoretic Extremal Functions. Potential Anal., 42(2):311–334, 2015.

[SZ99] B. Shiffman and S. Zelditch. Distribution of zeros of random and quantum chaotic sections of positive line bundles. Comm. Math. Phys., 200(3):661–683, 1999.

[SZ08] B. Shiffman and S. Zelditch. Number variance of random zeros on complex manifolds. Geom.Funct. Anal., 18(4):1422–1475, 2008.

Meshless Finite Difference Methods

Prof. Oleg Davydov[1]

[1] *Mathematisches Institut, Universität Giessen (Germania)*
Email: Oleg.Davydov@math.uni-giessen.de

Timetable: 16 hrs.
Scheduling of the Course:
Oct. 5,6,7,8 from 10:30 to 12:00
Oct. 12 from 10:30 to 12:00
Oct. 14 from 10:30 to 12:00 and from 02:30 to 04:00 PM
Oct. 15 from 10:30 to 12:00
the Course will be held online. The link Zoom to the event will be communicated as soon as possible.

Course requirements: Undergraduate Linear Algebra, Calculus and Numerical Analysis.

Examination and grading: a small project or an oral exam.

SSD: MAT/08 - MAT/05

Aim: Introduction in meshless Finite Difference methods for partial differential equations.

Course contents:

1. Introduction into positive definite functions and reproducing kernel Hilbert spaces. (2h)
2. Numerical differentiation with polynomials and kernels I-II. (4h)
3. Meshless finite difference methods: Introduction and error bounds I-II. (4h)
4. Meshless finite difference methods: Computational aspects I-III. (6h)

A smooth tour around rough models in finance (From data to stochastics to machine learning)

Prof. Antoine Jacquier[1]

[1] *Imperial College, Londra*
Email: a.jacquier@imperial.ac.uk

Timetable: 16 hrs. First lecture on March 30th, 2022, 16:00 (dates already fixed, see calendar on https://dottorato.math.unipd.it/calendar), Room 2BC30.

Course requirements: Probability and Stochastic Calculus

Examination and grading: oral examination on the topics covered during the course

SSD: MAT/06, SECS-S/06

Aim: Aim: the course aims at introducing the recent theory on rough volatility models, namely stochastic volatility models in finance driven by the fractional Brownian motion. This class of models will naturally arise by looking at market data and at the end of the course the PhD student will have full control of advanced tools in stochastic calculus which are crucial in modern finance.

Course contents:
A quick glance at time series in market data (Equities, Currencies, Commodities, Rates...) leaves no doubt that volatility is not deterministic over time, but stochastic. However, the classical Markovian setup, upon which a whole area of mathematical finance was built, was recently torn apart when Gatheral-Jaisson-Rosenbaum showed that the instantaneous volatility is not so well behaved and instead features memory and more erratic path behaviour. Rough volatility was born. This new paradigm does not come for free, though, and new tools and further analyses are needed in order to put forward the benefits of this new approach. The goal of this course is to explain how Rough Volatility naturally comes out of the data, and to study the new techniques required to use it as a tool for financial modelling. We shall endeavour to strike a balance between theoretical tools and practical examples, and between existing results and open problems. The contents shall span, with more or less emphasis on each topic, the following:

1. Estimating roughness from data. Constructing a rough volatility model.
2. Constructing a model consistent between the historical and the pricing measure: joint calibration of SPX and VIX options.
3. Pricing options in rough volatility models: from Hybrid Monte Carlo to Deep learning

The first item is anchored in fairly classical Statistics and Probability, while the second deals with Stochastic analysis. The last item draws upon recent literature connecting Path-dependent PDEs, Backward SDEs and Deep Learning technology. Prior knowledge in all areas is not required, but good Probability/Stochastic analysis background is essential.

Stochastic differential equations involving fractional Brownian motion

Prof. Yuliya Mishura[1]

[1] Taras Shevchenko National University of Kyiv
Department of Probability, Statistics and Actuarial Mathematics
Email: yuliyamishura@knu.ua

Timetable: 12 hrs. All lectures in Torre Archimede, Room 2BC/30 and online via Zoom. Lectures on July 8, 11, 12, 13, 14, 15, always at 14.30–16.30.

Course requirements: A previous knowledge of the basic concepts of stochastic processes is required. Knowledge of stochastic calculus could help for the advanced parts of the course, but during the course the basic concepts will be introduced for the understanding.

Examination and grading: Seminar.

SSD: MAT/06 Probability and Mathematical Statistics

Course contents:

The course will present following issues concerning stochastic differential equations with fractional Brownian motion: standard ones with smooth coefficients; the equations with unbounded coefficients and $H > 1/2$, the equations with unbounded coefficients and any H, together with the approximations and numerics; delayed and mixed equations. Some issues on discontinuous coefficients will be provided.

Standard stochastic differential equations with fBm. (2 hours)

1. Integration with respect to fBm.
2. Elements of fractional and fractional stochastic calculus.
3. Stochastic differential equations with fBm and smooth coefficients.

Equations with unbounded coefficients and $H > 1/2$ (2 hours):

1. Cox-Ingersoll-Ross equations with fBm.
2. Equations with unbounded drift.
3. Reflected Ornstein-Uhlenbeck equations.

“Sandwiched” equations (4 hours):

1. What drifts allow to consider low values of H in SDE.
2. Approximations of solutions.

Equations with delay, mixed equations and the case of discontinuous coefficients. (4h):

1. Equations with delay.
2. Mixed equations.
3. Discontinuous coefficients.

Topics in Stochastic Analysis

Prof. Tiziano Vargiolu[1]

[1] *Università di Padova*
Dipartimento di Matematica "Tullio Levi-Civita"
Email: vargiolu@math.unipd.it

Calendario: 8 hrs. **POSTPONED TO THE NEXT YEAR**

Prerequisiti: A previous knowledge of the basics of continuous time stochastic analysis with standard Brownian motion, i.e. stochastic integrals, Itô formula and stochastic differential equations, as given for example in the master course ”Analisi Stocastica”.

Tipologia di esame: Seminar

SSD: MAT/06

Programma: The program will be fixed with the audience according to its interests. Some examples could be:

- continuous time stochastic control;
- Levy processes;
- numerical methods;
- stochastic control.

Courses offered within the Master's Degree in Mathematics

(Courses included in this section will count up to 16 hours in the total amount of students'exams)

The Master Degree (Laurea Magistrale) in Mathematics of this Department offers many courses on a wide range of topics, in Italian or in English. The PhD students are encouraged to follow the parts of such courses they think are useful to complete their basic knowledge in Mathematics. In some cases this activity can receive credits from the Doctoral school, upon recommendation of the supervisor of the student. Since the courses at the Master level are usually less intense than those devoted to graduate students, the number of hours given as credits by our Doctorate will be less than the total duration of the course. Some examples of courses that receive such credits, unless the student already has the material in his background, are the following.

Topology 2

Prof. Andrea D'Agnolo
Università di Padova, Dipartimento di Matematica
Email: dagnolo@math.unipd.it

Period: 1st semester

Contents and other information:

https://didattica.unipd.it/off/2021/LM/SC/SC1172/001PD/SCQ0094298/N0

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Differential Equations

Prof. Martino Bardi
Università di Padova, Dipartimento di Matematica
Email: bardi@math.unipd.it

Period: 2nd semester

Contents and other information:

https://didattica.unipd.it/off/2021/LM/SC/SC1172/010PD/SCQ0093962/N0

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Homology and Cohomology

Prof. Bruno Chiarellotto
Università di Padova, Dipartimento di Matematica
Email: chiarbru@math.unipd.it

Period: 2nd semester

Contents and other information:

https://didattica.unipd.it/off/2021/LM/SC/SC1172/001PD/SCQ0094302/N0

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Calculus of Variations

Prof. Roberto Monti
Università di Padova, Dipartimento di Matematica
Email: monti@math.unipd.it

Period: 2nd semester

Contents and other information:

https://didattica.unipd.it/off/2021/LM/SC/SC1172/010PD/SCQ0093999/N0

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Hamiltonian Mechanics

Prof. Paolo Rossi
Università di Padova, Dipartimento di Matematica
Email: paolo.rossi.2@unipd.it

Period: 2nd semester

Contents and other information:

https://didattica.unipd.it/off/2021/LM/SC/SC1172/010PD/SCQ0094081/N0

Soft Skills

1. Maths information: retrieving, managing, evaluating, publishing SS-1
2. Writing a CV for academic positions in preparation
3. Writing a post doc application in preparation
4. Active participation in events organized by the Department devoted to the popularization of mathematics, like Venetonight, Kidsuniversity and others.

Doctoral Program in Mathematical Sciences

a.a. 2021/2022

SOFT SKILLS

Maths information: retrieving, managing, evaluating, publishing

Abstract: This course deals with the bibliographic databases and the resources provided by the University of Padova; citation databases and metrics for research evaluation; open access publishing and the submission of PhD theses and research data in UniPd institutional repositories.

Language: The Course will be held in Italian or in English according to the participants

Timetable: 5 hrs – October 27, 2021, 09:30 (2:30 hrs), November 3, 2021, 09:30 (2:30 hrs), the Course will be held online

Other courses suggested to the students

The students are encouraged to follow also courses outside Padova if they are useful for their training to research, in accordance with their supervisor. Parts of such course can be counted in fulfilment of their duties, provided the student passes an exam. The number of hours recognised as credits will be decided by the Coordinator after hearing the supervisor. Some examples of courses that receive such credits are the following.

Courses in collaboration with the Doctoral School on "Information Engineering"

(Courses included in this section will count up to 16 hours in the total amount of students'exams)

for complete Catalogue and class schedule see on

https://phd.dei.unipd.it/course-catalogues/

Calendar of activities on

`https://calendar.google.com/calendar/u/0/embed?src=fvsl9bgkbnhhkqp5mmqpiurn6c@group.calendar.google.com&ctz=Europe/Rome`

Bayesian Machine Learning

Giorgio Maria Di Nunzio[1]

[1] *Department of Information Engineering*
Email: dinunzio@dei.unipd.it

Timetable: 20 hrs. Lectures on Wednesday and Friday, 10:30-12:30, starting from March 9th, 2022 (see on https://phd.dei.unipd.it/course-catalogues/)

Course requirements: Basics of Probability Theory. Basics of R Programming.

Examination and grading: Homework assignments and final project.

SSD: Information Engineering

Aim: The course will introduce fundamental topics in Bayesian reasoning and how they apply to machine learning problems. In this course, we will present pros and cons of Bayesian approaches and we will develop a graphical tool to analyse the assumptions of these approaches in classical machine learning problems such as classification and regression.

Course contents:

Introduction of classical machine learning problems.

1. Mathematical framework
2. Supervised and unsupervised learning

Bayesian decision theory

1. Two-category classification
2. Minimum-error-rate classification
3. Bayes decision theory
4. Decision surfaces

Estimation

1. Maximum Likelihood Estimation
2. Expectation Maximization
3. Maximum A Posteriori
4. Bayesian approach

Graphical models

1. Bayesian networks
2. Two-dimensional visualization

Evaluation

1. Measures of accuracy

References:

1. J. Kruschke, Doing Bayesian Data Analysis: A Tutorial Introduction With R and Bugs, Academic Press 2010
2. Christopher M. Bishop, Pattern Recognition and Machine Learning (Information Science and Statistics), Springer 2007
3. Richard O. Duda, Peter E. Hart, David G. Stork, Pattern Classification (2nd Edition), Wiley-Interscience, 2000
4. Yaser S. Abu-Mostafa, Malik Magdon-Ismail, Hsuan-Tien Lin, Learning from Data, AML-Book, 2012 (supporting material available at http://amlbook.com/support.html)
5. David J. C. MacKay, Information Theory, Inference and Learning Algorithms, Cambridge University Press, 2003 (freely available and supporting material at http://www.inference.phy.cam.ac.uk/mackay/
6. David Barber, Bayesian Reasoning and Machine Learning, Cambridge University Press, 2012 (freely available at http://web4.cs.ucl.ac.uk/staff/D.Barber/pmwiki/pmwiki.php?n=
7. Kevin P. Murphy, Machine Learning: A Probabilistic Perspective, MIT Press, 2012 (supporting material http://www.cs.ubc.ca/ murphyk/MLbook/)
8. Richard McElreath, Statistical Rethinking, CRC Presso, 2015 (supporting material https://xcelab.net/rm/statistical-rethinking/)

Statistical Methods

Prof. Lorenzo Finesso[1]

[1] *CNR IEIIT Padova*
email: lorenzo.finesso@unipd.it

Timetable: 24 hrs. sLectures on Wednesday and Friday, 14:30-16:30, starting from May 4th, 2022 (see on https://phd.dei.unipd.it/course-catalogues/)

Enrollment: Students must enroll in the course using the Enrollment Form on the PhD Program eLearning platform (requires SSO authentication).

Course requirements: familiarity with basic linear algebra and probability.

Examination and grading: Homework assignments

SSD: Information Engineering

Aim: The course will present a small selection of statistical techniques which are widespread in applications. The unifying power of the information theoretic point of view will be stressed.

Course contents:

- *Background material.* The noiseless source coding theorem will be quickly reviewed in order to introduce the notions of entropy and informational divergence (relative entropy or Kullback-Leibler distance) between positive measures.
- *Divergence minimization problems.* Three I-divergence minimization problems will be posed and, via examples, connected with basic methods of statistical inference: ML (maximum likelihood), ME (maximum entropy), and EM (expectation-maximization).
- *Multivariate analysis methods.* The three standard multivariate methods, PCA (Principal Component Analysis), Factor Analysis, and CCA (Canonical Correlations Analysis) will be reviewed and their connection with divergence minimization discussed. Applications of PCA to least squares (PCR principal component regression, PLS Partial least squares). Approximate matrix factorization and PCA, with a brief detour on the approximate Nonnegative Matrix Factorization (NMF) problem.
- *EM methods.* The Expectation-Maximization method will be introduced in the context of Maximum Likelihood (ML) estimation with partial observations (incomplete data) and interpreted as an alternating divergence minimization algorithm à la Csiszár Tusnády.
- *Applications to stochastic processes.* Introduction to HMM (Hidden Markov Models). Maximum likelihood estimation for HMM via the EM method. If time allows: derivation of the Burg spectral estimation method as solution of a Maximum Entropy problem.

References:
Lecture notes and a list of references will be posted on the course moodle site.

Computational Inverse Problems

Prof. Fabio Marcuzzi[1]

[1] *Dipartimento di Matematica "Tullio Levi-Civita", Universitá Padova*
e-mail: marcuzzi@math.unipd.it

Timetable: 20 hrs. Lectures on Tuesday and Thursday, 14:30-16:30, starting from March 1st, 2022 (see on https://phd.dei.unipd.it/course-catalogues/)

Course requirements:
- basic notions of linear algebra and, possibly, numerical linear algebra.
- the examples and homework will be in Python (the transition from Matlab to Python is effortless).

Examination and grading: Homework assignments and final test.

SSD: MAT/08

Aim: We study numerical methods that are of fundamental importance in computational inverse problems. Real application examples will be given for distributed parameter systems in continuum mechanics. Computer implementation performance issues will be considered as well.

Course contents:

- definition of inverse problems, basic examples and numerical difficulties.
- numerical methods for QR and SVD and their application to the square-root implementation in PCA, least-squares, model reduction and Kalman filtering; recursive least-squares; High Performance Computing (HPC) implementation of numerical linear algebra algorithms.
- regularization methods;
- underdetermined linear estimation problems and sparse recovery;
- numerical algorithms for nonlinear parameter estimation: nonlinear least-squares (Levenberg-Marquardt), back-propagation learning;
- underdetermined nonlinear estimation problems and deep learning;
- examples with distributed parameter systems in continuum mechanics: reconstruction of forcing terms and parameters estimation;

References:

1 F.Marcuzzi "Computational Inverse Problems", lecture notes (will be posted on the moodle page of the course)

2 G. Strang, "Linear Algebra and Learning From Data", Wellesley - Cambridge Press, 2019

3 L. Trefethen and J. Bau, "Numerical Linear Algebra", SIAM, 1997

Applied Functional Analysis and Machine Learning

Prof. Gianluigi Pillonetto[1]

[1] Department of Information Engineering, Univ. Padova
e-mail: giapi@dei.unipd.it

Timetable: 28 hrs. Lectures on Tuesday and Thursday, 14:30-16:30, starting from Nov 16, 2021 (see on https://phd.dei.unipd.it/course-catalogues/)

Enrollment: students must enroll in the course using the Enrollment Form on the PhD Program eLearning platform (requires SSO authentication).

Course requirements: The classical theory of functions of real variable: limits and continuity, differentiation and Riemann integration, infinite series and uniform convergence. The arithmetic of complex numbers and the basic properties of the complex exponential function. Some elementary set theory. A bit of linear algebra.

Examination and grading: Homework assignments and final test.

SSD: Information Engineering

Aim: The course is intended to give a survey of the basic aspects of functional analysis, machine learning, regularization theory and inverse problems.

Course contents:

Review of some notions on metric spaces and Lebesgue integration: Metric spaces. Open sets, closed sets, neighborhoods. Convergence, Cauchy sequences, completeness. Completion of metric spaces. Review of the Lebesgue integration theory. Lebesgue spaces.

Banach and Hilbert spaces: Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded linear operators. Linear functionals. The finite dimensional case. Normed spaces of operators and the dual space. Weak topologies. Inner product spaces and Hilbert spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Representation of functionals on Hilbert spaces.

Compact linear operators on normed spaces and their spectrum: Spectral properties of bounded linear operators. Compact linear operators on normed spaces. Spectral properties of compact linear operators. Spectral properties of bounded self-adjoint operators, positive operators, opera- tors defined by a kernel. Mercer Kernels and Mercer theorem.

Reproducing kernel Hilbert spaces, inverse problems and regularization theory: Representer theorem. Reproducing Kernel Hilbert Spaces (RKHS): definition and basic properties. Examples of RKHS. Function estimation problems in RKHS. Tikhonov regularization. Primal and dual formulation of loss functions. Regularization networks. Consistency/generalization and relationship with Vapnik's theory and the concept of V-gamma dimension. Support vector regression and classification.

Heuristics for Mathematical Optimization

Prof. Domenico Salvagnin[1]

[1] *Department of Information Engineering, Padova*
email: dominiqs@gmail.com - domenico.salvagnin@unipd.it

Timetable: 20 hrs. Lectures on Wednesday and Friday, 14:30-16:30, starting from Feb 16, 2022 (see on https://phd.dei.unipd.it/course-catalogues/)

Enrollment: students must enroll in the course using the Enrollment Form on the PhD Program eLearning platform (requires SSO authentication).

Course requirements:
- Moderate programming skills (on a language of choice)
- Basics in linear/integer programming.

Examination and grading: Final programming project.

SSD: Information Engineering

Aim: Make the students familiar with the most common mathematical heuristic approaches to solve mathematical/combinatorial optimization problems. This includes general strategies like local search, genetic algorithms and heuristics based on mathematical models.

Course contents:
- Mathematical optimization problems (intro).
- Heuristics vs exact methods for optimization (intro).
- General principle of heuristic design (diversification, intensification, randomization).
- Local search-based approaches.
- Genetic/population based approaches.
- The subMIP paradigm.
- Applications to selected combinatorial optimization problems: TSP, QAP, facility location, scheduling.

References:

1. Gendreau, Potvin “Handbook of Metaheuristics”, 2010
2. Marti, Pardalos, Resende “Handbook of Heuristics”, 2018

Applied Linear Algebra

Prof. Luca Schenato[1]

[1] *Department of Information Engineering, University of Padova*
Email: schenato@dei.unipd.it

Timetable: 16 hours. Lectures on Monday and Wednesday, 10:30-12:30, starting from April 4th, 2022 (see on https://phd.dei.unipd.it/course-catalogues/)

Enrollment: students must enroll in the course using the Enrollment Form on the PhD Program eLearning platform (requires SSO authentication).

Course requirements:: A good working knowledge of basic notions of linear algebra as for example in [1]. Some proficiency in MATLAB.

Examination and grading: Grading is based on Homeworks, Written final exam, Short presentation based on a recent paper of Linear Algebra Algorithms for Big Data.

Aim: We study concepts and techniques of linear algebra that are important for applications with special emphasis on the topics: solution of systems of linear equations with particular attention to the analysis of the backward error and computational cost of the basic algorithms and matrix equation. A wide range of exercises and problems will be an essential part of the course and constitute homework required to the student.

Objectives:

- Theory: formal proofs of many results (theorem-proof type problems)
- Algorithms: understanding of most commonly used algortithm used in MATLAB and Python for Linear Algebra
- Implementation: MATLAB implementation of algorithms and performance evaluation on Big Data

Course contents:

1. Vectors: inner products, norms, main operations (average, standard deviation, ...)
2. Matrices: matrix-vector and matrix-matrix multiplication, Frobenius norm,
3. Complexity, sparsity
4. Special matrices: Diagonal, Upper Triangular, Lower triangular, Permutation (general pair), inverse and orthogonal
5. A square and invertible: LU decomposition (aka gaussian elimination), LU-P decomposition, Cholesky decomposition
6. Ax=b via LU-P decomposition: forward and backward substitution
7. (sub)Vector spaces: definitions, span, bases (standard, orthogonal, orthonormal), dimension, direct sum, orthogonal complement, null space, orthogonal complement theorem
8. Gram-Smith orthogonalization and QR decomposition (square and invertible A, general non-square)

9. Ax=b via QR decomposition. LU-P vs QR
10. Linear maps: image space, kernel, column and row rank
11. Fundamental Theorem of Linear Algebra (Part I): rank-nullity Theorem, the 4 fundamental subspace
12. Eigenvalues/eigenvector and Shur decomposition
13. Projection matrices: oblique and orthogonal, properties
14. Positive semidefinite matrices: properties and quadratic functions square root matrix
15. Properties of A'A and AA' and Polar decomposition
16. Singular Value Decomposition: proofs and properties
17. Pseudo-inverse: definition and relation to SVD
18. Fundamental Theorem of Linear Algebra (Part II): special orthogonal basis for diagonalization
19. Least-Squares: definition, solution and algorithms
20. Ill-conditioned problems vs stability of algorithms, numerical conditioning of algorithms, numerical conditionings

References:

[1] S. Boyd, L. Vanderberghe, "Introduction to Applied Linear Algebra", Cambridge University Press, 2018

[2] G. Strang, "The Fundamental Theorem of Linear Algebra", The American Mathematical Monthly, vol. 100(9), pp. 848-855, 1993

[3] G. Strang, "Linear Algebra and Learning From Data", Wellesley - Cambridge Press, 2019

Elements of Deep Learning

Prof. Gian Antonio Susto[1]

[1] *Department of Information Engineering, Univ. Padova*
e-mail: gianantonio.susto@dei.unipd.it

Timetable: 24 hrs. Lectures on Tuesday and Thursday, 10:30-12:30, starting from Jan 18, 2022 (see on https://phd.dei.unipd.it/course-catalogues/)

Enrollment: students must enroll in the course using the Enrollment Form on the PhD Program eLearning platform (requires SSO authentication).

Course requirements: Basics of Machine Learning and Python Programming.

Examination and grading: Final project.

SSD: Information Engineering

Aim: The course will serve as an introduction to Deep Learning (DL) for students who already have a basic knowledge of Machine Learning. The course will move from the fundamental architectures (e.g. CNN and RNN) to hot topics in Deep Learning research.

Course contents:

- Introduction to Deep Learning: context, historical perspective, differences with respect to classic Machine Learning.
- Feedforward Neural Networks (stochastic gradient descent and optimization).
- Convolutional Neural Networks.
- Neural Networks for Sequence Learning.
- Elements of Deep Natural Language Processing.
- Elements of Deep Reinforcement Learning.
- Unsupervised Learning: Generative Adversarial Neural Networks and Autoencoders.
- Laboratory sessions in Colab.
- Hot topics in current research.

References:

1. Arjovsky, M., Chintala, S., Bottou, L. (2017). Wasserstein GAN. CoRR, abs/1701.07875.
2. Bahdanau, D., Cho, K., Bengio, Y. (2014). Neural Machine Translation by Jointly Learning to Align and Translate. CoRR, abs/1409.0473.
3. I. Goodfellow, Y. Bengio, A. Courville 'Deep Learning', MIT Press, 2016
4. Goodfellow, I.J., Pouget-Abadie, J., Mirza, M., Xu, B., Warde-Farley, D., Ozair, S., Courville, A.C., Bengio, Y. (2014). Generative Adversarial Nets. NIPS.
5. Hochreiter, S., Schmidhuber, J. (1997). Long Short-Term Memory. Neural computation, 9 8, 1735-80.

6. Kalchbrenner, N., Grefenstette, E., Blunsom, P. (2014). A Convolutional Neural Network for Modelling Sentences. ACL.
7. Krizhevsky, A., Sutskever, I., Hinton, G.E. (2012). ImageNet Classification with Deep Convolutional Neural Networks. Commun. ACM, 60, 84-90.
8. LeCun, Y. (1998). Gradient-based Learning Applied to Document Recognition.
9. Mikolov, T., Sutskever, I., Chen, K. (2013). Representations of Words and Phrases and their Compositionality.
10. Vincent, P., Larochelle, H., Lajoie, I., Bengio, Y., Manzagol, P. (2010). Stacked Denoising Autoencoders: Learning Useful Representations in a Deep Network with a Local Denoising Criterion. Journal of Machine Learning Research, 11, 3371-3408.
11. Zaremba, W., Sutskever, I., Vinyals, O. (2014). Recurrent Neural Network Regularization. CoRR, abs/1409.2329.