Categorical aspects of realizability

Samuele Maschio

1 Dipartimento di Matematica “Tullio Levi-Civita” - Università di Padova
Email: maschio@math.unipd.it

Timetable: 12 hrs. First lecture on November 4th, 2021, 14:00 (dates already fixed, see Calendar of the Activities at https://dottorato.math.unipd.it/calendar), Torre Archimede, Room 2BC/30

Course requirements: Basic notions in category theory and logic.

Examination and grading: Examination will consist of a seminar based on research papers on the topic of the course.

SSD: MAT/01, MAT/02, MAT/04, INF/01

Aim: The aim of the course is to present the categorical account of realizability. Realizability was introduced by Kleene in 1945 in order to provide an interpretation of intuitionistic arithmetics. The notion of realizability is connected to that of computability and exploits crucially Universal Turing Machine Theorem. As shown starting from the 80s, realizability can be presented in a categorical setting as it was done by Hyland with his Effective Topos, which provides a categorical framework for constructive mathematics. Here we take into account this construction in the wider context of tripos theory and exact completions, and we will present some of the main properties of realizability toposes and their internal mathematics.

Course contents: Kleene realizability. Elementary toposes; first-order hyperdoctrines; triposes; tripos-to-topos construction; partial combinatory algebras; realizability toposes and the effective topos; arithmetics in the effective topos; set theory in the effective topos; regular and exact categories; ex/reg completion; the category of assemblies, ex/lex completion; the category of partitioned assemblies.

Bibliography: