Optimal Transport and Wasserstein Gradient Flows

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Timetable: 24 hrs in the second term. The course will include a part given in Padua and a part which is given online (approx 50%-50%). Precise details and schedule TBA

Course requirements: some functional analysis (for instances chapters 1, 3, 4, 8 and 9 of Brezis' book on functional analysis) and some notions of basic PDEs. However, the main required notions will be recalled during the course.

Examination and grading: Oral examination (on the content of the course, or presentation of a related research paper/subject, according to the preferences of each student)

SSD: MAT-05

Aim: With the first part of the course students will learn the main features of the theory of optimal transport; the second part will allow them to master more specialized tools from this theory in their applications to some evolution PDEs with a gradient flow structure

Course contents:
Monge and Kantorovich problems, existence of optimal plans, duality.
Existence of optimal maps, Brenier’s theorem (optimal maps in the quadratic case are gradient of convex functions), connection with the Monge-Ampèere equation.
Optimal transport for the distance cost. Wasserstein distances and their properties.
Curves in the Wasserstein spaces and relation with the continuity equation. Characterization of AC curves in the Wasserstein spaces
Geodesics in the Wasserstein spaces. Dynamic Benamou-Brenier formulation.
Introduction to gradient flows in metric spaces. The JKO minimization scheme for some parabolic equation.
Convergence of the JKO scheme for the Heat and Fokker-Planck equations.
Regularity estimates from the JKO scheme (Lipschitz, BV, Sobolev . . . ).

Bibliography: