Optimal Transport and Wasserstein Gradient Flows

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Timetable: 24 hrs. First lecture on April 11th, 2022, 09:00.

Location: The course will be held in presence and online according with the following calendar:
Apr 11, 12, 09:00-11:00, in Room 1BC45 (Prof. Davide Vittone),
plus lectures that will take place in 10 out of the following 12 slots:
Apr 20, 21, 10:30-12:30 and 14:00-16:00, in Room 2BC30, Apr 22, 10:30-12:30, in Room 2AB40, and 14:00-16:00, in Room 1BC45 (Prof. Filippo Santambrogio)
Apr 26, May 3, 10, 17, 24, 31 14:00-16:00 online (Prof. Filippo Santambrogio)

Course requirements: some functional analysis (for instances chapters 1, 3, 4, 8 and 9 of Brezis' book on functional analysis) and some notions of basic PDEs. However, the main required notions will be recalled during the course.

Examination and grading: Oral examination (on the content of the course, or presentation of a related research paper/subject, according to the preferences of each student)

SSD: MAT-05

Aim: With the first part of the course students will learn the main features of the theory of optimal transport; the second part will allow them to master more specialized tools from this theory in their applications to some evolution PDEs with a gradient flow structure

Course contents:
Monge and Kantorovich problems, existence of optimal plans, duality.
Existence of optimal maps, Brenier's theorem (optimal maps in the quadratic case are gradient of convex functions), connection with the Monge-Ampère equation.
Optimal transport for the distance cost. Wasserstein distances and their properties.
Curves in the Wasserstein spaces and relation with the continuity equation. Characterization of AC curves in the Wasserstein spaces
Geodesics in the Wasserstein spaces. Dynamic Benamou-Brenier formulation.
Introduction to gradient flows in metric spaces. The JKO minimization scheme for some parabolic equation.
Convergence of the JKO scheme for the Heat and Fokker-Planck equations.
Regularity estimates from the JKO scheme (Lipschitz, BV, Sobolev…).

Bibliography:
F. Santambrogio: *Optimal Transport for Applied Mathematicians*, Birkhauser (2015)
C. Villani: *Topics in Optimal Transportation,* American Mathematical Society (2003)
L. Ambrosio, N. Gigli, G. Savaré: *Gradient Flows in Metric Spaces and in the Space of Probability Measures*, Birkhauser (2005)
F. Santambrogio: {Euclidean, Metric, and Wasserstein} Gradient Flows: an overview, *Bulletin of Mathematical Sciences*, available online (2017).