Applied Linear Algebra

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Timetable: 16 hours. Lectures on Monday and Wednesday, 10:30-12:30, starting from April 4th, 2022 (see on https://phd.dei.unipd.it/course-catalogues/)

Enrollment: students must enroll in the course using the Enrollment Form on the PhD Program eLearning platform (requires SSO authentication).

Course requirements: A good working knowledge of basic notions of linear algebra as for example in [1]. Some proficiency in MATLAB.

Examination and grading: Grading is based on Homeworks, Written final exam, Short presentation based on a recent paper of Linear Algebra Algorithms for Big Data.

Aim: We study concepts and techniques of linear algebra that are important for applications with special emphasis on the topics: solution of systems of linear equations with particular attention to the analysis of the backward error and computational cost of the basic algorithms and matrix equation. A wide range of exercises and problems will be an essential part of the course and constitute homework required to the student.

Objectives:

• Theory: formal proofs of many results (theorem-proof type problems)
• Algorithms: understanding of most commonly used algorithm used in MATLAB and Python for Linear Algebra
• Implementation: MATLAB implementation of algorithms and performance evaluation on Big Data

Course contents:

1. Vectors: inner products, norms, main operations (average, standard deviation, ...)
2. Matrices: matrix-vector and matrix-matrix multiplication, Frobenius norm
3. Complexity, sparsity
4. Special matrices: Diagonal, Upper Triangular, Lower triangular, Permutation (general pair), inverse and orthogonal
5. A square and invertible: LU decomposition (aka gaussian elimination), LU-P decomposition, Cholesky decomposition
6. Ax=b via LU-P decomposition: forward and backward substitution
7. (sub)Vector spaces: definitions, span, bases (standard, orthogonal, orthonormal), dimension, direct sum, orthogonal complement, null space, orthogonal complement theorem
8. Gram-Smith orthogonalization and QR decomposition (square and invertible A, general non-square)
9. Ax=b via QR decomposition. LU-P vs QR
10. Linear maps: image space, kernel, column and row rank
11. Fundamental Theorem of Linear Algebra (Part I): rank-nullity Theorem, the 4 fundamental subspace
12. Eigenvalues/eigenvector and Shur decomposition
13. Projection matrices: oblique and orthogonal, properties
14. Positive semidefinite matrices: properties and quadratic functions square root matrix
15. Properties of A’A and AA’ and Polar decomposition
16. Singular Value Decomposition: proofs and properties
17. Pseudo-inverse: definition and relation to SVD
18. Fundamental Theorem of Linear Algebra (Part II): special orthogonal basis for diagonalization
19. Least-Squares: definition, solution and algorithms
20. Ill-conditioned problems vs stability of algorithms, numerical conditioning of algorithms, numerical conditionings

References: