Qualitative and quantitative properties for elliptic equations

Alessandro Goffi

Università degli Studi di Padova, Dipartimento di Matematica
Email:alessandro.goffi@unipd.it

Timetable: 16 hrs. First lecture on September 26, 2022, 11:00 (date already fixed, see the Calendar of activities at https://dottorato.math.unipd.it/calendar) Torre Archimede, Room 2BC30.

Course requirements: Basic knowledge of linear elliptic PDEs, Sobolev and Hölder spaces.

Examination and grading: The exam will be oral and tailored on the basis of the students’ interests.

SSD: MAT/05

Aim: Introduce some classical and modern methods to study qualitative and quantitative properties for elliptic problems, such as Liouville and regularity theorems.

Course contents:

- Basic concepts in the theory of partial differential equations (PDEs): quick review on the classification of linear and nonlinear equations, various notions of solutions and useful function spaces;
- Review of some basic tools for harmonic, sub- and superharmonic functions. Mean-value properties, strong and weak maximum principles, Harnack inequalities, Caccioppoli inequalities, Bernstein-type gradient bounds and some consequences: Hölder regularity, Liouville-type theorems under various a priori conditions on the solution ($L^\infty$, $L^p$, finite Dirichlet energy, one-side bounds). Characterization of harmonic functions through touching functions: the notion of viscosity solution;
- Nonlinear equations (mostly driven by the Laplacian):
  - PDEs with gradient dependent terms:
    * Hölder/Lipschitz regularity for semi-solutions/solutions via integral/maximum principle methods;
    * Liouville properties for elliptic equations and inequalities via maximum principle and integral methods respectively.
  - PDEs with zero-th order nonlinearities:
    * Liouville properties for solutions using integral approaches and for semi-solutions via maximum principle methods.
  - If time permits, some discussions and extensions to problems driven by p-Laplacian, mean-curvature, fully nonlinear second order operators.
Bibliography:


