# Applied Functional Analysis and Machine Learning: from regularization to deep networks

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**Timetable:** 28 hrs (see Class Schedule on https://phd.dei.unipd.it/course-catalogues/)

**Enrollment:** add the course to the list of courses you plan to attend using the Course Enrollment Form (requires SSO authentication) and, if you are taking the course for credits, to the Study and Research Plan.

**Course requirements:** The classical theory of functions of real variable: limits and continuity, differentiation and Riemann integration, infinite series and uniform convergence. The arithmetic of complex numbers and the basic properties of the complex exponential function. Some elementary set theory. A bit of linear algebra.

**Examination and grading:** Homework assignments and final test.

**SSD:** Information Engineering

**Aim:** The course is intended to give a survey of the basic aspects of functional analysis, machine learning, regularization theory and inverse problems.

**Course contents:**
Review of some notions on metric spaces and Lebesgue integration: Metric spaces. Open sets, closed sets, neighborhoods. Convergence, Cauchy sequences, completeness. Completion of metric spaces. Review of the Lebesgue integration theory. Lebesgue spaces.
Banach and Hilbert spaces: Finite dimensional normed spaces and subspaces. Compactness and finite dimension. Bounded linear operators. Linear functionals. The finite dimensional case. Normed spaces of operators and the dual space. Weak topologies. Inner product spaces and Hilbert spaces. Orthogonal complements and direct sums. Orthonormal sets and sequences. Representation of functionals on Hilbert spaces.
Reproducing kernel Hilbert spaces, inverse problems and regularization theory: Representer theorem. Reproducing Kernel Hilbert Spaces (RKHS): definition and basic properties. Examples of RKHS. Function estimation problems in RKHS. Tikhonov regularization. Support vector regression and classification. Extensions of the theory to deep kernel-based networks: multi-valued RKHSs and the concatenated Representer Theorem.

**References:**

1. G. Pillonetto, T. Chen, A. Chiuso, G. De Nicolao, L. Ljung. Regularized System Identification – learning dynamic models from data, Springer Nature 2022
2. W. Rudin. Real and Complex Analysis, McGraw Hill, 2006
3. C.E. Rasmussen and C.K.I. Williams. Gaussian Processes for Machine Learning. The MIT Press, 2006
4. H. Brezis, Functional analysis, Sobolev spaces and partial differential equations, Springer 2010