Reading course:
Torsion pairs in abelian categories

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Timetable: 24 hrs. First lecture on 25/11/2022, 16.00, alternating between Torre Archimede, Room TBA (Padova) or Ca'Vignal 2, Room TBA (Verona), with a zoom link for the participants outside the institution in which the lecture takes place. There will be regular meetings following the 25/11/2022: those dates will be discussed in the first meeting. A schedule of speakers, and related references will also be discussed during the first meeting. Interested participants are invited to contact the organisers beforehand.

Course requirements: Participants in this reading seminar should be familiar with some elementary aspects on rings, modules, categories and homological algebra. Complementary materials may be provided upon request to cover any specific topic for which participants feel they need further preparation.

Examination and grading: Participants are expected to attend the lectures in a participative way and to deliver at least one lecture covering part of the program.

SSD: MAT/02

Aim: A torsion pair in an abelian category is a decomposition of it into two parts: a torsion and a torsionfree part. It turns out that both the study of individual torsion pairs and the study of the whole collection of torsion pairs often provides us with useful information on the category. This is a recurring technique both in representation theory and in algebraic geometry (see also the course of Anna Barbieri in our PhD School).

In this reading course, structured through a series of seminars, we aim to provide participants with a wide range of techniques and examples concerning the study of torsion pairs in abelian categories. The course will focus both on categorical aspects as well as on two particular contexts where examples are well-understood: in the representations theory of finite-dimensional algebras and in the study of modules over commutative noetherian rings.

Course contents:

1. Grothendieck categories and finiteness properties. Torsion pairs in abelian categories and closure properties. Hereditary torsion pairs and categorical localisations.
2. Elements of approximation theory. Functorially finite torsion classes. Elements of support τ-tilting theory. τ-tilting finiteness.
3. Torsion pairs in length categories, lattice structure, brick labelling, completely join/meet irreducible torsion classes, semi-bricks, mono-bricks.

4. Definability and torsion pairs.
5. Classification of torsion pairs using geometric models for finite-dimensional algebras: type A, tube categories, cluster-tilted type A-tilde.
6. Torsion pairs for commutative noetherian rings: Matlis's correspondence, support and Gabriel's theorem.

If time allows we may discuss research directions and further recent developments, which may include silting/cosilting theory and torsion pairs in triangulated categories.

Bibliography:

1. T.Adachi, O.Iyama, I.Reiten, *τ-tilting theory*, Comp. Math. **150**(3) (2014), 415–452.
2. L.Angeleri Hügel, *On the abundance of silting modules*, Surveys in representation theory of algebras, Contemp. Math. **716**, Amer. Math. Soc., Providence, RI (2018), 1-23.
3. K. Bauer, A. Buan, R. Marsh, *Torsion pairs and rigid objects in tubes*, Alg. Rep. Th. **17**(2) (2014), 565-591.
4. W. Crawley-Boevey, *Locally finitely presented additive categories*, Comm. Algebra **22**(5) (1994), 1641–1674.
5. P. Gabriel, *Des catégories abéliennes*, Bull. Soc. Math. France **90** (1962), p. 323-448.
6. B. Stenström, *Rings of quotients*, Springer, 1975.